

Campfire queen Cycling champion Sentimental geologist*

Learn more about
Marjon Walrod
and tell us more
about you. Visit
pwc.com/bringit.

Your life. You can
bring it with you.



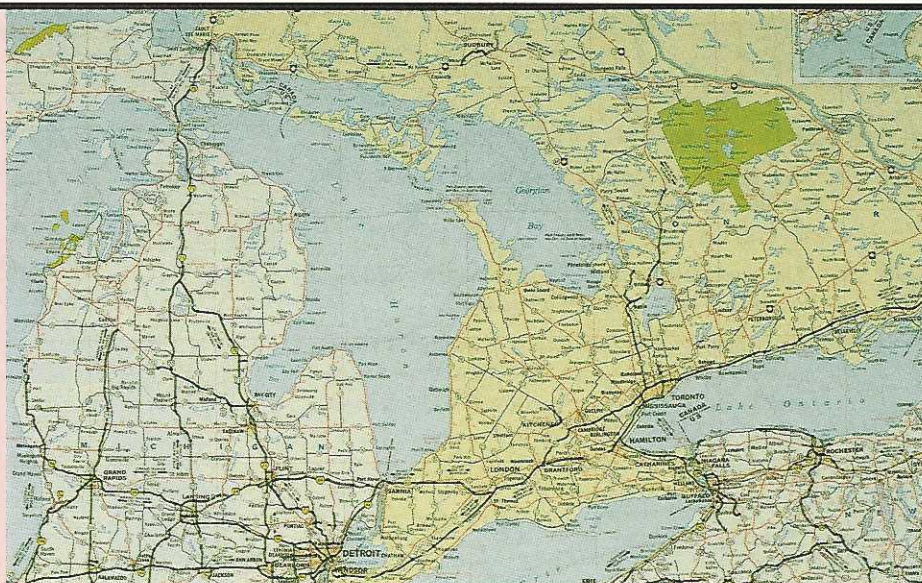
*connectedthinking

PRICEWATERHOUSECOOPERS 

© 2006 PricewaterhouseCoopers LLP. All rights reserved. "PricewaterhouseCoopers" refers to PricewaterhouseCoopers LLP (a Delaware limited liability partnership) or, as the context requires, the PricewaterhouseCoopers global network or other member firms of the network, each of which is a separate and independent legal entity. *connectedthinking is a trademark of PricewaterhouseCoopers LLP (US). We are proud to be an Affirmative Action and Equal Opportunity Employer.

Rational Expressions, Ratio and Proportion

On a road map, 3 inches represents a distance of 45 miles. If the distance between Detroit and Sault Ste. Marie is 23 inches on the map, how far is it from Detroit to Sault Ste. Marie?



5-1 ■ Rational numbers and rational expressions

A rational expression

In chapter 1, we defined a rational number.

Rational numbers

A rational number is a number that can be written as the quotient of two integers with the divisor (denominator) not zero.

Examples of rational numbers are

$$6, \quad \frac{3}{4}, \quad \frac{-5}{6}, \quad \text{and} \quad \frac{8}{7}$$

We extend this definition to involve the quotient of two polynomials and define a **rational expression**.

Rational expressions

A **rational expression** is an expression of the form

$$\frac{P}{Q}$$

where P and Q are polynomials, $Q \neq 0$.

Concept

A rational expression is an expression that can be written as the quotient of two polynomials with the denominator not zero.

For example,

$$\frac{2x}{x+1}, \quad \frac{x^2-2}{x^2-x-6}, \quad \text{and} \quad \frac{x^2+x}{5}$$

are all rational expressions.

Just as a rational number has a numerator and a denominator, so does a rational expression. In the rational expression

$$\frac{x^2-2}{x^2-x-6}$$

the polynomial on the top, $x^2 - 2$, is called the **numerator** and the polynomial on the bottom, $x^2 - x - 6$, is called the **denominator**.

Note Any polynomial is a rational expression since the denominator can be considered to be 1. For example, $x^2 - 2 = \frac{x^2 - 2}{1}$.

Evaluating a rational expression

In chapter 2, we evaluated algebraic expressions by substituting given values for the variables and performing the indicated operations. We follow the same procedure when evaluating rational expressions.

■ Example 5-1 A

Evaluate the following rational expressions for the given value of the variable.

1. $\frac{5x-2}{4x+3}, x=2$

$$\begin{aligned} \frac{5x-2}{4x+3} &= \frac{5(2)-2}{4(2)+3} \\ &= \frac{10-2}{8+3} \\ &= \frac{8}{11} \end{aligned}$$

Replace x with 2

Perform indicated operations using order of operations

2. $\frac{x+2}{x^2-3x-10}, x=5$

$$\begin{aligned} \frac{x+2}{x^2-3x-10} &= \frac{5+2}{(5)^2-3(5)-10} \\ &= \frac{7}{25-15-10} \\ &= \frac{7}{0} \quad (\text{undefined}) \end{aligned}$$

Replace x with 5

Perform indicated operations

Note Anytime zero is in the denominator, the expression is undefined. Therefore, for the rational expression to be defined, x cannot be 5.

$$3. \frac{x-5}{2x^2+x-1}, x = -\frac{1}{2}$$

$$\frac{x-5}{2x^2+x-1} = \frac{\left(-\frac{1}{2}\right) - 5}{2\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 1}$$

Replace x with $-\frac{1}{2}$

$$= \frac{-\frac{1}{2} - 5}{\frac{1}{2} - \frac{1}{2} - 1}$$

Perform indicated operations

$$= \frac{-\frac{11}{2}}{-1}$$

$$= \frac{11}{2}$$

$$= \frac{11}{2}$$

Multiply numerator and denominator by -1

► **Quick check** Evaluate: $\frac{3y-2}{y+3}$ when $y = 4$ and $\frac{y^2-y-1}{y^2-4}$ when $y = -2$ ■

Domain of a rational expression

Notice in example 2, the answer was undefined since division by zero is not defined. The rational expression becomes *meaningless* for those values of the variable for which the denominator equals zero, as with $x = 5$ in example 2. Finding the value(s) of the variable that will make the denominator zero is called *finding the restrictions on the variable(s)*. All other values of the variable for which the expression is defined make up the **domain** of the rational expression.

Domain of a rational expression

The set of all replacement values of the variable for which a rational expression is defined determines the **domain** of the rational expression.

In finding the domain of a rational expression, we use the following procedure.

Finding the domain of a rational expression

1. Factor the denominator into a product of prime polynomials, if possible.
2. Set each factor of the denominator containing the variable equal to zero (using the zero product property).
3. Solve the resulting equations. The solutions are the restrictions placed on the variable.

■ Example 5-1 B

Determine the domain of each of the following rational expressions.

$$1. \frac{a-3}{a-4}$$

$$a-4=0$$

$$a=4$$

Set denominator equal to 0

Solve equation for a

The restriction is that $a \neq 4$. Domain is all real numbers except 4.

Note We look *only* at the denominator. The value(s) of the variable for which the numerator is zero is of no concern to us.

$$2. \frac{3x^2}{x^2-x-6}$$

$$\frac{3x^2}{x^2-x-6} = \frac{3x^2}{(x-3)(x+2)}$$

$$x-3=0 \quad \text{or} \quad x+2=0$$

$$x=3$$

$$x=-2$$

Factor the denominator

Set each factor equal to 0

Solve each equation for x

The restrictions are $x \neq 3$ or $x \neq -2$. Domain is all real numbers except 3 or -2.

$$3. \frac{x+3}{x^2+4}$$

x^2+4 does not factor. If there is a restriction, it will occur when x^2 is -4. Since x^2 is never negative, the sum x^2+4 can never be zero and there are no restrictions on the variable. Thus, the domain is the set of all real numbers.

$$4. \frac{x-3}{x^2-x}$$

$$\frac{x-3}{x^2-x} = \frac{x-3}{x(x-1)}$$

$$x=0 \quad \text{or} \quad x-1=0$$

$$x=0$$

$$x=1$$

Factor the denominator

Set each factor equal to 0

Solve each equation for x

The restrictions are $x \neq 0$ or $x \neq 1$. Domain is all real numbers except 0 or 1.

Note In example 4, a common error is to forget to place restrictions on the factor x .

► **Quick check** Determine the domain: $\frac{x+3}{x^2+x-6}$

Mastery points

Can you

- Evaluate a rational expression for a given value of the variable?
- Determine the restrictions on the variable in a rational expression?
- Determine the domain of a rational expression?

Exercise 5-1

Evaluate each given rational expression using the given value of the variable. If the given value is not in the domain of the rational expression, so state. See example 5-1 A.

Examples $\frac{3y - 2}{y + 3}; y = 4$

Solutions $\frac{3(4) - 2}{(4) + 3}$
 $\frac{12 - 2}{4 + 3}$
 $\frac{10}{7}$

Replace y with 4

Perform indicated operations

$\frac{y^2 + 2y - 1}{y^2 - 4}; y = -2$

$\frac{(-2)^2 + 2(-2) - 1}{(-2)^2 - 4}$

Replace y with -2

$\frac{4 - 4 - 1}{4 - 4}$

Perform indicated operations

$\frac{-1}{0}$ (Undefined)

-2 is not in the domain.

1. $\frac{x}{3x}; x = 2$

2. $\frac{2x + 1}{5x - 3}; x = 3$

3. $\frac{5a^2 + 2}{a - 1}; a = 1$

4. $\frac{x^2 - 1}{x}; x = 0$

5. $\frac{-4p^2}{2p - 3}; p = -1$

6. $\frac{-5b^3}{5 - 2b}; b = -2$

7. $\frac{x + 9}{x^2 + 2x - 1}; x = 4$

8. $\frac{2n - 3}{3n^2 + n - 1}; n = -3$

9. $\frac{(-2x)^2}{x^2 + 3x + 7}; x = 2$

10. $\frac{(-x)^3}{2x^2 - 5}; x = 3$

11. $\frac{3x^2 + 2x + 1}{4 - x - x^2}; x = 1$

12. $\frac{8 - b - 2b^2}{4 - 3b^2}; b = -2$

13. $\frac{3 - 4x}{x^2 - x}; x = 0$

14. $\frac{x^3 - 2x^2 + x}{x - 2}; x = \frac{1}{2}$

15. $\frac{-x + 3}{x^2 + 2x + 1}; x = -\frac{1}{3}$

Determine the domain of the given rational expression. See example 5-1 B.

Example $\frac{x + 3}{x^2 + x - 6}$

Solution $\frac{x + 3}{(x + 3)(x - 2)}$

Factor the denominator

$x + 3 = 0$ or $x - 2 = 0$
 $x = -3$ $x = 2$

Set each factor equal to zero
 Solve for x

The restrictions are $x \neq -3$ or $x \neq 2$ and the domain is the set of all real numbers except -3 or 2 .

16. $\frac{4}{3x}$

17. $\frac{5}{4x}$

18. $\frac{8}{x - 2}$

19. $\frac{10}{x - 5}$

20. $\frac{x}{x + 7}$

21. $\frac{3x^2}{x + 3}$

22. $\frac{x + 1}{2x - 1}$

23. $\frac{a + 9}{4a - 3}$

24. $\frac{p - 3}{5 - 2p}$

25. $\frac{y + 4}{8 - 3y}$

26. $\frac{x + 7}{x^2 + 3x - 18}$

27. $\frac{8b + 1}{b^2 - 7b + 6}$

28. $\frac{5s^2 + 7}{2s^2 - s - 3}$

29. $\frac{8z}{3z^2 + 2z - 8}$

30. $\frac{4}{x^2 - 4}$

$$\begin{array}{lllll}
 31. \frac{5x}{9x^2 - 4} & 32. \frac{a - 2}{4a^2 - 16} & 33. \frac{b + 3}{5b^2 - 45} & 34. \frac{7x}{3x - 15} & 35. \frac{5b - 1}{9b - 21} \\
 36. \frac{16x}{8x^2 - 18} & 37. \frac{17q}{3q^2 - 3} & 38. \frac{23}{x^2 + 1} & 39. \frac{5x - 3}{2x^2 + 1} & 40. \frac{2x^2}{x^2 + 9}
 \end{array}$$

41. In business, the acid-test ratio (A) is given by $A = \frac{C + R}{L}$, where C is the cash on hand, R represents receivables, and L is the current liability. For what value of L is A not defined?
42. In college algebra, the sum S of the terms in some infinite geometric sequence is given by $S = \frac{a}{1 - r}$, where a is the first term and r is the common ratio. For what value of r is S not defined?
43. In physics, the *ideal gas law* of a constant mass of gas is given by $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, where P_1 = the pressure in the first state, V_1 = the volume in the first state, T_1 = the temperature in Kelvin degrees in the first state, P_2 = the pressure in the second state, V_2 = the volume in the second state, and T_2 = the temperature in Kelvin degrees in the second state. What restrictions on the variables must be placed in this formula?

Review exercises

- The statement $4(5 + 6) = 4 \cdot 5 + 4 \cdot 6$ demonstrates what property of real numbers? See section 1-8.
- Simplify the expression $2[3 - 4(1 + 6)]$ by performing the indicated operations. See section 1-8.
- Evaluate the expression $a[b + (c - d)]$ when $a = 1$, $b = -2$, $c = 3$, and $d = -4$. See section 2-2.
- Reduce the fraction $\frac{35}{63}$ to lowest terms. See section 1-1.

Factor the following expressions. See section 4-3.

- $2x^2 - 9x - 5$
- $4y^2 - 40y + 100$

5-2 ■ Simplifying rational expressions

The fundamental principle of rational expressions

One of the most important procedures we can use when we work with rational expressions is the simplification of the rational expression. To do this, we use a principle called the **fundamental principle of rational expressions**.

Fundamental principle of rational expressions

If P is any polynomial and Q and R are nonzero polynomials, then

$$\frac{PR}{QR} = \frac{P}{Q} \text{ and } \frac{P}{Q} = \frac{PR}{QR}$$

Concept

To change the appearance of a rational expression without changing its value, we may multiply or divide both the numerator and the denominator by the same nonzero polynomial.

This property is based on 1 being the identity element for multiplication. That is,

$$\frac{PR}{QR} = \frac{P}{Q} \cdot \frac{R}{R} = \frac{P}{Q} \cdot 1 = \frac{P}{Q}$$

This property permits us to **reduce** rational expressions to *lowest terms*. A rational expression is *completely reduced* if the greatest factor common to both the numerator and the denominator is 1 or -1 . We can see that the key to reducing rational expressions is *finding* and dividing out *factors* that are common to both the numerator and the denominator.

To reduce a rational expression

1. Write the numerator and the denominator in factored form.
2. Divide the numerator and the denominator by all common factors.

■ Example 5-2 A

Simplify the following rational expressions by reducing to lowest terms. Assume that all denominators are nonzero.

$$\begin{aligned} 1. \quad \frac{45}{60} &= \frac{3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5} && \text{Factor numerator and denominator to prime factors} \\ &= \frac{3 \cdot (3 \cdot 5)}{2 \cdot 2 \cdot (3 \cdot 5)} && \text{Group common factors } (3 \cdot 5) \\ &= \frac{3}{2 \cdot 2} && \text{Divide numerator and denominator by } (3 \cdot 5) \\ &= \frac{3}{4} && \text{Multiply remaining factors} \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{14x^2}{10x^3} &= \frac{7 \cdot 2 \cdot x \cdot x}{5 \cdot 2 \cdot x \cdot x \cdot x} && \text{Factor numerator and denominator} \\ &= \frac{7 \cdot (2 \cdot x \cdot x)}{5 \cdot x \cdot (2 \cdot x \cdot x)} && \text{Group common factors } (2 \cdot x \cdot x) \\ &= \frac{7}{5x} && \text{Divide numerator and denominator by common factor } (2 \cdot x \cdot x) \end{aligned}$$

$$\begin{aligned} 3. \quad \frac{5a - 15}{4a - 12} &= \frac{5(a - 3)}{4(a - 3)} && \text{Factor numerator and denominator} \\ &= \frac{5}{4} && \text{Divide numerator and denominator by common factor } (a - 3) \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{y - 7}{y^2 - 49} &= \frac{y - 7}{(y + 7)(y - 7)} && \text{Factor denominator} \\ &= \frac{1}{y + 7} && \text{Divide numerator and denominator by common factor } (y - 7) \end{aligned}$$

► **Quick check** Reduce $\frac{25z^4}{15z^5}$ and $\frac{a^2 - 36}{a^2 - a - 30}$ to lowest terms. ■

Note This reducing process is often called “cancelling” the common factors. It is important to remember that we are *dividing* both the numerator and the denominator by the same common factor; we are *not* just “crossing out” quantities. This leads us to our next topic.

Common errors when reducing

The fundamental property allows us to divide by common *factors only*. A common error in example 4 is to divide the numerator and the denominator by y and 7. These are *terms* and this cannot be done. For example,

$$\frac{9}{11} = \frac{8+1}{8+3} \neq \frac{\cancel{8}+1}{\cancel{8}+3} = \frac{1}{3}$$

This error can be avoided by always remembering that the *fundamental principle of rational expressions* allows us to *divide* the numerator and the denominator by common factors. y and 7 are *terms* and *not factors*.

Reducing $\frac{a-b}{b-a}$

Consider the rational expression $\frac{x-5}{5-x}$, which does not appear to be reducible by a common factor. However,

$$\begin{aligned} 5-x &= -1(-5+x) && \text{Factor out } -1 \\ &= -1(x-5) && \text{Commute the terms and use the} \\ &&& \text{definition of subtraction} \end{aligned}$$

Thus,

$$\begin{aligned} \frac{x-5}{5-x} &= \frac{x-5}{-1(x-5)} \\ &= \frac{1}{-1} && \text{Reduce by common factor } (x-5) \\ &= -1 \end{aligned}$$

In general, for all real numbers a and b , $a \neq b$,

$$\frac{a-b}{b-a} = -1$$

■ Example 5-2 B

Simplify the following rational expressions by reducing to lowest terms. Assume that no denominator equals zero.

$$\begin{aligned} 1. \quad & \frac{4-x}{x^2-16} \\ &= \frac{4-x}{(x-4)(x+4)} && \text{Completely factor the denominator} \\ &= \frac{4-x}{x-4} \cdot \frac{1}{x+4} && \text{Factor opposites } 4-x \text{ and } x-4 \\ &= -1 \cdot \frac{1}{x+4} && \frac{4-x}{x-4} = -1 \\ &= \frac{-1}{x+4} && \text{Multiply numerator by } -1 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{1-x^2}{2x^2+x-3} \\
 &= \frac{(1-x)(1+x)}{(x-1)(2x+3)} && \text{Completely factor numerator and denominator} \\
 &= \frac{1-x}{x-1} \cdot \frac{1+x}{2x+3} && \text{Factor opposites } 1-x \text{ and } x-1 \\
 &= -1 \cdot \frac{1+x}{2x+3} && \frac{1-x}{x-1} = -1 \\
 &= \frac{-1(1+x)}{2x+3} && \text{Multiply numerator by } -1 \\
 &= \frac{-1-x}{2x+3} \quad \text{or} \quad \frac{-x-1}{2x+3} && \text{Alternative forms of answer}
 \end{aligned}$$

► **Quick check** Reduce $\frac{16-y^2}{3y^2-11y-4}$ to lowest terms. ■

Mastery points

Can you

- Reduce a rational expression to lowest terms using the fundamental principle of rational expressions?
- Recognize factors $a-b$ and $b-a$ and use $\frac{a-b}{b-a} = -1$?

Exercise 5-2

Simplify the following rational expressions by reducing to lowest terms. Assume that no denominator equals zero. See example 5-2 A.

Examples $\frac{25z^4}{15z^5}$

Solutions
$$\begin{aligned}
 &= \frac{5 \cdot 5 \cdot z \cdot z \cdot z \cdot z}{5 \cdot 3 \cdot z \cdot z \cdot z \cdot z \cdot z} \\
 &= \frac{5 \cdot (5 \cdot z \cdot z \cdot z \cdot z)}{3 \cdot z \cdot (5 \cdot z \cdot z \cdot z \cdot z)} \\
 &= \frac{5}{3z}
 \end{aligned}$$

Factor numerator and denominator
Group common factors
($5 \cdot z \cdot z \cdot z \cdot z$)
Divide numerator and denominator by
($5 \cdot z \cdot z \cdot z \cdot z$)

$$\frac{a^2-36}{a^2-a-30}$$

$$\begin{aligned}
 &= \frac{(a+6)(a-6)}{(a+5)(a-6)} \\
 &= \frac{a+6}{a+5}
 \end{aligned}$$

Factor numerator and denominator
Divide numerator and denominator by
($a-6$)

1. $\frac{54}{72}$

2. $\frac{75}{145}$

3. $\frac{6x}{15}$

4. $\frac{8a}{10}$

5. $\frac{16x^2}{12x}$

6. $\frac{15b^3}{20b}$

7. $\frac{-8x^2}{6x^4}$

8. $\frac{3a^6}{-9a^3}$

9. $\frac{16a^2b}{20ab^2}$

10. $\frac{15a^2x^3}{35ax^2}$

11. $\frac{20ab^2c^3}{-4ab^2c^3}$

12. $\frac{-72x^4y^3z^2}{9x^4y^3z^2}$

13. $\frac{10(x+5)}{8(x+5)}$

14. $\frac{24(x-3)}{15(x-3)}$

15. $\frac{6(x-2)}{(x+3)(x-2)}$

16. $\frac{-8(x+1)}{4(x+1)(x-6)}$

17. $\frac{a+b}{a^2-b^2}$

21. $\frac{3x-3}{6x+6}$

25. $\frac{x^2-3x-10}{x^2-x-6}$

29. $\frac{x-3}{x^3-27}$

18. $\frac{x^2-y^2}{x-y}$

22. $\frac{6y-6}{8y^2-8}$

26. $\frac{y^2-y-42}{y^2+12y+36}$

30. $\frac{x+2}{x^3+8}$

19. $\frac{3m-6}{5m-10}$

23. $\frac{x^2-9}{x^2+6x+9}$

27. $\frac{2y^2-3y-9}{4y^2-13y+3}$

31. $\frac{a^2-b^2}{a^3+b^3}$

20. $\frac{8b+12}{10b+15}$

24. $\frac{a^2-10a+25}{a^2-25}$

28. $\frac{4m^2-15m-4}{8m^2-18m-5}$

32. $\frac{x^3+y^3}{x^2-y^2}$

Simplify by reducing to lowest terms. Assume that no denominator is equal to zero. See example 5-2 B.

Example $\frac{16-y^2}{3y^2-11y-4}$

Solution $= \frac{(4-y)(4+y)}{(y-4)(3y+1)}$

Factor numerator and denominator

$$= \frac{4-y}{y-4} \cdot \frac{4+y}{3y+1}$$

$$= -1 \cdot \frac{4+y}{3y+1}$$

$$\frac{4-y}{y-4} = -1$$

$$= \frac{-1(4+y)}{3y+1}$$

Multiply the numerator by -1

$$= \frac{-4-y}{3y+1} \quad \text{or} \quad \frac{-y-4}{3y+1}$$

Alternative forms of answer

33. $\frac{4x-4y}{y-x}$

34. $\frac{8b-8a}{a-b}$

35. $\frac{2x-8}{12-3x}$

36. $\frac{12a-8b}{10b-15a}$

37. $\frac{2y^2-2x^2}{x-y}$

38. $\frac{3p-3q}{6q^2-6p^2}$

39. $\frac{(x-y)^2}{y^2-x^2}$

40. $\frac{a-b}{b^2-a^2}$

41. $\frac{n^2-m^2}{(m+n)^2}$

42. $\frac{p^2-q^2}{q^2-p^2}$

43. $\frac{4x-4y}{y^2-x^2}$

44. $\frac{4-y}{2y^2-7y-4}$

45. $\frac{x-3}{12-x-x^2}$

Review exercises

1. Write the decimal number 0.000314 in scientific notation. See section 3-5.

2. Given $x = 2$, $y = -3$ and $z = -1$, evaluate the expression $\frac{4x-y}{2y+z}$. See sections 2-2 and 5-1.

3. A piece of lumber 16 feet long is to be divided into two pieces so that one piece is 1 foot longer than twice the length of the other piece. Find the lengths of the two pieces of lumber. See section 2-8.

Simplify the following expressions. Assume all denominators are nonzero. Express answers with positive exponents only. See section 3-4.

4. $(5x^{-2}y^3)^0$

5. $\frac{3xy^2}{3^{-2}xy^{-1}}$

6. $(-2x^3y^2)^3$

Love The Taste. Taste The Love.

At Culver's® we can't think of anything better than serving up our creamy frozen custard and delicious classics cooked fresh the minute you order them. Which is why when we bring them to your table, they're always accompanied by a warm smile and a friendly offer to see if there's anything else we can get for you. So come on into your neighborhood Culver's and see for yourself. You might just be in love by the time you leave.



5-3 ■ The quotient of two polynomials

In section 3-3, we observed the process of dividing a monomial by a monomial. We shall first review this process before we deal with other types of polynomial division. Recall the quotient property for division of expressions that have like bases, $a^m \div a^n = a^{m-n}$, $a \neq 0$.

■ Example 5-3 A

Find the indicated quotients. Assume all variables are nonzero.

1. $x^7 \div x^4 = x^{7-4}$ Subtract exponents when dividing
 $= x^3$
2. $\frac{2^2 a^5}{2a^3} = 2^{2-1} a^{5-3}$ Divide like bases by subtracting exponents
 $= 2^1 a^2$ Perform indicated subtractions
 $= 2a^2$ $2^1 = 2$

► **Quick check** Find the quotient of $\frac{5^3 x^4}{5x}$

Division of a polynomial by a monomial

Consider the indicated division.

$$\frac{3x^3 - 9x^2 + 15x}{3x}$$

To perform this division, we use a principle of fractions.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad (c \neq 0)$$

By reversing this equation, the principle can be used to divide a polynomial by a monomial.

Division of a polynomial by a monomial

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad (c \neq 0)$$

Concept

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

■ Example 5-3 B

Find the indicated quotients. Assume all denominators are nonzero.

1. $\frac{3x^3 - 9x^2 + 15x}{3x} = \frac{3x^3}{3x} - \frac{9x^2}{3x} + \frac{15x}{3x}$ Divide each term of the numerator by the monomial denominator
 $= \frac{3}{3}x^{3-1} - \frac{9}{3}x^{2-1} + \frac{15}{3}x^{1-1}$ Quotient property of exponents
 $= 1 \cdot x^2 - 3 \cdot x^1 + 5 \cdot x^0$ Subtract and divide as indicated
 $= x^2 - 3x + 5 \cdot 1$ $x^0 = 1$
 $= x^2 - 3x + 5$

$$2. \frac{8a^4 + 4a^2 - 12a}{4a} = \frac{8a^4}{4a} + \frac{4a^2}{4a} - \frac{12a}{4a}$$

$$= 2a^3 + a - 3$$

Divide each term of numerator by the monomial denominator
Simplify each term by reducing

$$3. \frac{5a^7 + 15a^5 - 10a}{5a^2} = \frac{5a^7}{5a^2} + \frac{15a^5}{5a^2} - \frac{10a}{5a^2}$$

$$= a^5 + 3a^3 - \frac{2}{a}$$

Divide each term of numerator by the monomial denominator
Simplify each term by reducing

Recall that we can check our division by
(quotient)(divisor) = dividend

In example 2,

$$(2a^3 + a - 3)(4a) = 2a^3 \cdot 4a + a \cdot 4a - 3 \cdot 4a$$

$$= 8a^4 + 4a^2 - 12a$$

Distributive property
Dividend

► **Quick check** Find the quotient: $\frac{16x^5 + 20x^3 - 4x^2}{4x^2}$

Note A common error in this type of problem is demonstrated in the following example.

$$\frac{x^3 + x^2}{x^2} \neq \frac{x^3 + 1}{1}$$

It is tempting to simply "cancel" the x^2 in the numerator with the x^2 in the denominator, but the correct procedure would be

$$\frac{x^3 + x^2}{x^2} = \frac{x^3}{x^2} + \frac{x^2}{x^2} = x + 1$$

Remember that the entire numerator and the entire denominator must be divided by the same quantity. In the example, only *part* of the numerator was divided by x^2 . *Only factors* may be divided by factors.

Division of a polynomial by a polynomial

Consider a quotient in which the divisor is not a monomial. For example,

$$\frac{y^2 - y - 2}{y - 2}$$

← Dividend
← Divisor

which involves the division of a trinomial by a binomial. We handle this just like long division with numbers. Set it up in the form

$$y - 2 \overline{) y^2 - y - 2}$$

Note The divisor and dividend must be arranged in descending powers of one variable with zeros inserted to hold the position of any missing term.

The following table demonstrates writing a polynomial in descending powers of the variable and inserting zeros to hold the position of missing terms.

Dividend	Dividend arranged in descending powers
$x^3 + 2x + 3x^4 + 4x^2 - 1$	$3x^4 + x^3 + 4x^2 + 2x - 1$
$x^3 + x - 9$	$x^3 + 0x^2 + x - 9$
$x^4 - 1$	$x^4 + 0x^3 + 0x^2 + 0x - 1$

The method for dividing polynomials is similar to the long division used in dividing whole numbers. To demonstrate this, we divide 972 by 36 step-by-step as we divide $(y^2 - y - 2)$ by $(y - 2)$.

$36 \overline{)972}$	$y - 2 \overline{)y^2 - y - 2}$
Step 1 Divide 36 into 97, which goes 2 times. Place 2 over 7 in the dividend.	Divide y into y^2 , which goes y times. Place y over y in the dividend.
$\begin{array}{r} 2 \\ 36 \overline{)972} \end{array}$	$\begin{array}{r} y \\ y - 2 \overline{)y^2 - y - 2} \end{array}$
Step 2 Multiply 2 times 36, place 72 below 97 in the dividend.	Multiply y times $(y - 2)$, place $y^2 - 2y$ below $y^2 - y$ in the dividend.
$\begin{array}{r} 2 \\ 36 \overline{)972} \\ \underline{72} \end{array}$	$\begin{array}{r} y \\ y - 2 \overline{)y^2 - y - 2} \\ \underline{y^2 - 2y} \end{array}$
Step 3 Subtract 72 from 97. The difference is 25.	Subtract $y^2 - 2y$ from $y^2 - y$. $(y^2 - y) - (y^2 - 2y) =$ $y^2 - y - y^2 + 2y = y$
$\begin{array}{r} 2 \\ 36 \overline{)972} \\ (-) \underline{72} \\ 25 \end{array}$	$\begin{array}{r} y \\ y - 2 \overline{)y^2 - y - 2} \\ (-) \underline{y^2 - 2y} \\ y \end{array}$ Change signs and add
Step 4 Bring down the next digit of the dividend, 2.	Bring down the next term of the dividend, -2 .
$\begin{array}{r} 2 \\ 36 \overline{)972} \\ \underline{72} \\ 252 \end{array}$	$\begin{array}{r} y \\ y - 2 \overline{)y^2 - y - 2} \\ \underline{y^2 - 2y} \\ y - 2 \end{array}$
Step 5 Divide 36 into 252, which goes 7 times. Place 7 over 2 in the dividend.	Divide y into y , which goes 1 time. Place 1 over 2 in the dividend with a plus sign between y and 1.
$\begin{array}{r} 27 \\ 36 \overline{)972} \\ \underline{72} \\ 252 \end{array}$	$\begin{array}{r} y + 1 \\ y - 2 \overline{)y^2 - y - 2} \\ \underline{y^2 - 2y} \\ y - 2 \end{array}$

Step 6 Multiply 7 times 36, which is 252. Place this product below 252 at the bottom.

$$\begin{array}{r} 27 \\ 36 \overline{)972} \\ \underline{72} \\ 252 \\ (-) \underline{252} \\ 0 \end{array}$$

Multiply 1 times $(y - 2)$, which is $y - 2$. Place this below $y - 2$ at the bottom.

$$\begin{array}{r} y + 1 \\ y - 2 \overline{)y^2 - y - 2} \\ \underline{y^2 - 2y} \\ y - 2 \\ (-) \underline{y - 2} \text{Change signs} \\ 0 \text{and add} \end{array}$$

Step 7 Subtract $252 - 252 = 0$. There is no remainder.

$$972 \div 36 = 27$$

Subtract $(y - 2) - (y - 2) = 0$.

$$\begin{aligned} (y^2 - y - 2) \div (y - 2) \\ = y + 1 \end{aligned}$$

Step 8 Check your division by multiplying the quotient by the divisor to see if you get the original dividend.

$$27 \cdot 36 = 972$$

$$(y + 1)(y - 2) = y^2 - y - 2$$

Note A common error is committed when we subtract polynomials as we did in step 3. Remember, to subtract two polynomials, *change the signs of the second polynomial and then add*.

$$\begin{array}{r} y^2 - y \rightarrow y^2 - y \\ (-) y^2 - 2y \rightarrow -y^2 + 2y \\ \hline 0 + y = y \end{array}$$

The large majority of errors in this type of problem occur when polynomials are subtracted.

Example 5-3 C

Find the indicated quotient and check the answer.

$$\begin{array}{l} 1. \frac{x^2 + 3x - 4}{x + 4} \quad x + 4 \overline{)x^2 + 3x - 4} \\ \quad \quad \quad \underline{x^2 + 4x} \\ \quad \quad \quad -x - 4 \\ \quad \quad \quad \underline{-x - 4} \\ \quad \quad \quad 0 \end{array} \quad \begin{array}{l} x(x + 4) = x^2 + 4x \\ \text{Subtract to get } -x \text{ and bring down } -4 \\ -1(x + 4) = -x - 4 \\ \text{Subtract to get } 0 \end{array}$$

$$\text{Therefore, } \frac{x^2 + 3x - 4}{x + 4} = x - 1.$$

$$\begin{aligned} \text{Check: } (x - 1)(x + 4) &= x^2 + 4x - x - 4 \\ &= x^2 + 3x - 4 \end{aligned}$$

If we still have a remainder after “bringing down” all of the terms of the dividend, handle it as follows:

$$\begin{array}{l} 2. \frac{a^2 + 5a + 6}{a - 2} \quad a - 2 \overline{)a^2 + 5a + 6} \\ \quad \quad \quad \underline{a^2 - 2a} \\ \quad \quad \quad 7a + 6 \\ \quad \quad \quad \underline{7a - 14} \\ \quad \quad \quad 20 \end{array} \quad \begin{array}{l} a(a - 2) = a^2 - 2a \\ \text{Subtract to get } 7a. \text{ Bring down } 6 \\ 7(a - 2) = 7a - 14 \\ (7a + 6) - (7a - 14) = 7a + 6 - 7a \\ + 14 = 20 \end{array}$$

Hence, $\frac{a^2 + 5a + 6}{a - 2} = a + 7 + \frac{20}{a - 2}$, where the remainder 20 is placed over the divisor $a - 2$.

To check our answer, we add the remainder of 20 to the product of $(a + 7)$ and $(a - 2)$.

$$\begin{aligned}(a + 7)(a - 2) + (20) &= a^2 - 2a + 7a - 14 + (20) \\ &= a^2 + 5a + 6\end{aligned}$$

3. $\frac{x^3 - x + 2}{x - 3}$

Note that there is no term in the dividend that contains x^2 . The division will be easier to perform if the term $0x^2$ is inserted as a placeholder so that all powers of the variable x are present in descending order. Thus, we have

$$\frac{x^3 + 0x^2 - x + 2}{x - 3}$$

and the value of the dividend has not been changed since we have added $0x^2$, which is 0. Therefore, to perform the division, we get

$$\begin{array}{r}x^2 + 3x + 8 \\ x - 3 \overline{) x^3 + 0x^2 - x + 2} \\ \underline{(-) x^3 - 3x^2} \\ 3x^2 - x \\ \underline{(-) 3x^2 - 9x} \\ 8x + 2 \\ \underline{(-) 8x - 24} \\ 26\end{array}$$

$$\frac{x^3 - x + 2}{x - 3} = x^2 + 3x + 8 + \frac{26}{x - 3}$$

$$\begin{aligned}\text{Check: } (x^2 + 3x + 8)(x - 3) + (26) \\ &= x^3 - 3x^2 + 3x^2 - 9x + 8x - 24 + 26 \\ &= x^3 - x + 2\end{aligned}$$

Note When performing division, always count the number of terms in the denominator. If there is only one term, *do not use long division*.

► **Quick check** Find the quotient: $\frac{6x^2 - 7x - 3}{2x - 3}$

Mastery points

Can you

- Divide a monomial by a monomial?
- Divide a polynomial by a monomial?
- Divide a polynomial by a polynomial?
- Check the answer?

Exercise 5-3

Perform the indicated divisions and check the answers. See example 5-3 A.

Example $\frac{5^3x^4}{5x}$

Solution $= 5^3 - 1x^4 - 1$
 $= 5^2x^3$
 $= 25x^3$

Divide like bases by subtracting exponents
 Perform subtractions
 $5^2 = 25$

1. $\frac{8x^3}{2x}$
2. $\frac{-15x^5}{3x^2}$
3. $\frac{-65x^4y^2z}{13xy}$
4. $\frac{-28a^3b}{-7ab}$
5. $\frac{3(a-b)^2}{a-b}$
6. $\frac{5(x+y)^3}{x+y}$
7. $\frac{6a^2(b-c)^2}{3a(b-c)}$
8. $\frac{-10x^3(y-z)^3}{2x^2(y-z)}$
9. $\frac{12a^3b^2c(x+y)^3}{-3abc(x+y)^2}$

See example 5-3 B.

Example $\frac{16x^5 + 20x^3 - 4x^2}{4x^2}$

Solution $= \frac{16x^5}{4x^2} + \frac{20x^3}{4x^2} - \frac{4x^2}{4x^2}$
 $= 4x^3 + 5x - 1$

Divide denominator into each term of numerator
 Divide constants and apply properties of exponents

10. $\frac{6x-9}{3}$
11. $\frac{24a^2-12a}{-6}$
12. $\frac{bx^2-bx}{bx}$
13. $\frac{a^3-3a^2+2a}{a}$
14. $\frac{12x^3-8x^2+3x}{4x}$
15. $\frac{15a^3-9a^2+12a-6}{3a}$
16. $\frac{13a-a^2b^2+a^2b}{a^2b}$
17. $\frac{x^2y-xy^2-2xy^3}{-xy}$
18. $\frac{14a^2b^3-21a^2b^2-28ab}{7ab}$
19. $\frac{30x^3y^4+21x^2y^2-18x^2y^4}{3x^2y^2}$
20. $\frac{-21m^2n^5+35m^3n^2-14m^2n^2}{-7m^2n^2}$
21. $\frac{a(b-1)-c(b-1)}{b-1}$
22. $\frac{a(x-y)-b(x-y)}{x-y}$

See example 5-3 C.

Example $\frac{6x^2-7x-3}{2x-3}$

Solution
$$\begin{array}{r} 3x+1 \\ 2x-3 \overline{) 6x^2-7x-3} \\ \underline{(-) 6x^2-9x} \\ 2x-3 \\ \underline{(-) 2x-3} \\ 0 \end{array}$$

$3x(2x-3) = 6x^2-9x$
 Subtract to get $2x$ and bring down -3
 $1(2x-3) = 2x-3$
 $(2x-3)-(2x-3) = 2x-3-2x+3 = 0$

$$\frac{6x^2-7x-3}{2x-3} = 3x+1$$

The check is left to the student.

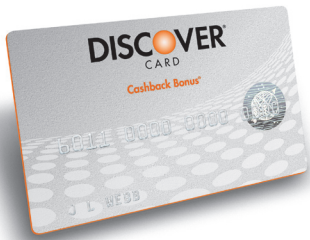
23. $\frac{a^2 + 7a + 10}{a - 2}$ 24. $\frac{x^2 + 8x + 15}{x + 5}$ 25. $\frac{a^2 + 5a + 10}{a + 3}$
26. $\frac{x^2 - x - 72}{x + 8}$ 27. $(a^2 + 6a + 10) \div (a + 3)$ 28. $(4a^2 + 1 + 4a) \div (2a + 1)$
29. $(9a^2 - 24a + 12) \div (3a - 4)$ 30. $(27a^3 - 1) \div (3a - 1)$ 31. $(x^3 - 8) \div (x - 2)$
32. $(x^4 - 14) \div (x - 2)$ 33. $\frac{x^3 + 4x^2 + 7x + 6}{x + 2}$ 34. $\frac{2a^3 - 3a^2 - 13a + 12}{a - 5}$
35. $\frac{b^3 + 6b^2 + 7b - 8}{b - 1}$ 36. $\frac{6x^4 - x^3 - 2x^2 - 7x - 19}{2x - 3}$
37. $(15a^2 + 28a - 32) \div (5a - 4)$ 38. $(x^4 - 2x^3 + 4x^2 - x + 3) \div (x^2 - x + 4)$
39. $(x^4 + 3x^3 - 6x^2 + 3x - 8) \div (x^2 + 3x - 5)$ 40. $(y^4 + 2y^3 - 4y + 2) \div (y^2 - y + 1)$
41. $(y^4 + 2y - 3) \div (y^2 + 2y - 5)$
42. A contractor uses the expression $x^2 + 6x + 8$ to represent the area of the floor of a room. If she decides that the length of the room will be represented by $x + 4$, what will the width of the room be in terms of x ?
43. An electrician uses the expression $4x^2 + 11x + 6$ to determine the amount of wire to order when wiring a house. If the expression comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $x + 2$, find the number of outlets in terms of x .
44. What polynomial when divided by $3x - 2$ yields the quotient $2x^2 + 3x - 5$?
45. What polynomial when divided by $-2x + 5$ yields the quotient $3x^3 - 2x + 6$?

Review exercises

- Find the solution set of the quadratic equation $4y^2 + 9y + 2 = 0$. See section 4-7.
- The area of a rectangle is 42 square feet. If the length is 1 foot longer than the width, what are the dimensions of the rectangle? See section 4-8.

Find the following products. See section 3-2.

- $(4x - 3)^2$
- $(x + 2)(x^2 + x - 1)$
- $(5y - 1)(5y + 1)$
- Reduce $\frac{3y^2 - 5y - 2}{2y^2 - y - 6}$ to lowest terms.
See section 5-2.



Extra Credit Rocks

Sign up for a Discover® Student Card today and enjoy:

- 0% Intro APR* on Purchases for 6 Months
- No Annual Fee
- Easiest Online Account Management Options
- Full 5% *Cashback Bonus*®* on Get More purchases in popular categories all year
- Up to 1% *Cashback Bonus*®* on all your other purchases
- Unlimited cash rewards that never expire as long as you use your Card

APPLY NOW

DISCOVER
CARD

*View Discover® Card Rates, Fees, Rewards and Other Important Information.

5-4 ■ Ratio and proportion

A ratio

We learned the fraction $\frac{a}{b}$ represents the indicated quotient of a divided by b . A **ratio** compares two numbers, or quantities, in the same way.

Ratio

A ratio is the comparison of two numbers (or quantities) by division.

The ratio of the number a to the number b is written

$$a \text{ to } b, \quad \frac{a}{b}, \quad \text{or} \quad a : b$$

We read $a : b$ as “the ratio of a to b ,” where a and b are called the *terms* of the ratio. The first number given is always the numerator and the second number is the denominator of the fraction representing the ratio.

■ Example 5-4 A

Write each ratio statement in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms.

1. The ratio of 3 to 4

$$3 : 4$$

Form $a : b$

$$\frac{3}{4}$$

Written as a fraction

2. The ratio of 15 to 9

$$15 : 9 \text{ or } 5 : 3$$

Form $a : b$ (divide 15 and 9 by the common factor 3)

$$\frac{15}{9} = \frac{5}{3}$$

Written as a fraction reduced to lowest terms

3. The ratio of $2\frac{1}{2}$ to $3\frac{1}{4}$

We first write each mixed number as an improper fraction.

$$2\frac{1}{2} = \frac{5}{2} \text{ and } 3\frac{1}{4} = \frac{13}{4}$$

$$\frac{2\frac{1}{2}}{3\frac{1}{4}} = \frac{5}{2} \div \frac{13}{4} = \frac{5}{2} \cdot \frac{4}{13} = \frac{10}{13}$$

Therefore, the ratio $2\frac{1}{2} : 3\frac{1}{4}$ becomes $10 : 13$ or $\frac{10}{13}$.

► **Quick check** Write the ratio of 18 to 21 in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms.

If the quantities have the same unit of measure, the ratio will be expressed by a fraction without any unit designation required.

4. 45 minutes to 60 minutes

$$\begin{aligned} 45 \text{ min} : 60 \text{ min} &= \frac{45 \text{ min}}{60 \text{ min}} && \text{Write as fraction } \frac{a}{b} \\ &= \frac{3}{4} \text{ or } 3 : 4 && \text{Reduce by dividing each term by 15} \end{aligned}$$

When the compared quantities are not of the same unit of measure but *can be stated* in the same unit, it may be desirable to do so. The ratio again becomes only a fraction, as in example 4.

5. 3 feet to 4 inches

Since 1 ft = 12 in., then 3 ft = 36 in., we have

$$\begin{aligned} 36 \text{ in.} : 4 \text{ in.} &= \frac{36 \text{ in.}}{4 \text{ in.}} && \text{Write as fraction } \frac{a}{b} \\ &= \frac{9}{1} \text{ or } 9 : 1 && \text{Reduce by dividing each term by 4} \end{aligned}$$

Note We reduced to $\frac{9}{1}$ to demonstrate the comparison that is present.

6. 35 cents to 4 dollars

Since there are 400 cents in 4 dollars, we have

$$\begin{aligned} 35 \text{ cents} : 400 \text{ cents} &= \frac{35 \text{ cents}}{400 \text{ cents}} && \text{Write as a fraction } \frac{a}{b} \\ &= \frac{7}{80} \text{ or } 7 : 80 && \text{Reduce by dividing each term by 5} \end{aligned}$$

Note When we change to a common unit of measure, it is easiest to change to the *smaller* unit of measure, as we did in the previous examples. Changing to the larger unit of measure usually involves fractions that are more difficult to reduce.

► **Quick check** Write the ratio of 16 minutes to 2 hours in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms.

Ratios are used to indicate relationships in many areas of the physical world.

1. The geographer makes maps and prints to scale, 20 miles to 1 inch.
2. The physicist measures air pressure and uses force per unit of area.

$$14.7 \text{ lb/in.}^2 = \frac{14.7 \text{ lb}}{1 \text{ in.}^2}$$

3. The auto mechanic interprets engine specifications by compression ratio, 9 to 1 or 9 : 1.
4. The machinist is concerned with gear ratio, 2 to 1 or 2 : 1.

When you compare two measurable quantities by ratio, it is not necessary for them to have the same unit of measure. If the units are not the same, you *must include the units* when you are expressing the ratio. These ratios represent rates of change.

■ Example 5-4 B

Express the following as a ratio in lowest terms.

1. 50 miles to 1 inch

$$50 \text{ mi} : 1 \text{ in.} = \frac{50 \text{ mi}}{1 \text{ in.}} \text{ or } 50 \text{ miles per inch}$$

2. 350 miles to 7 hours

$$\begin{aligned} 350 \text{ mi} : 7 \text{ hr} &= \frac{350 \text{ mi}}{7 \text{ hr}} && \text{Write as a fraction } \frac{a}{b} \\ &= \frac{50 \text{ mi}}{1 \text{ hr}} \text{ (stated 50 miles per hour)} && \text{Reduce by dividing each term by 7} \end{aligned}$$

► **Quick check** Express 64 pounds to 8 square inches as a ratio in lowest terms.

A proportion

A **proportion** establishes a relationship between two ratios.

Definition of a proportion

A **proportion** is a statement of equality of two ratios.

Given the ratios a to b and c to d ,

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d$$

is a proportion. We read the statement $a : b = c : d$ “ a is to b as c is to d .” The numbers a , b , c , and d are called the *terms* of the proportion.

$$\text{Given the proportion } \frac{a}{b} = \frac{c}{d},$$

$$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d} \quad \text{Multiply each member by } bd$$

$$ad = bc \quad \text{Reduce by } b \text{ on the left and by } d \text{ on the right}$$

Property of proportions

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc \text{ (} b, d \neq 0 \text{)}$$

Note The products ad and bc are found by multiplying diagonally.

$$\begin{array}{ccc} a & & c \\ & \searrow & \nearrow \\ b & & d \end{array} \quad \begin{array}{l} bc \\ ad \end{array}$$

This process is frequently called *cross-multiplying*, especially by persons in applied fields, and ad and bc are called the *cross products*.

Example 5-4 C

Determine if the following statements form a proportion.

$$1. \quad \frac{3}{5} = \frac{12}{20}$$

Using the property of proportions, we obtain

$$5 \cdot 12 = 60 \quad \text{and} \quad 3 \cdot 20 = 60$$

The cross products are both 60, so we have a proportion.

$$2. \quad \frac{5}{6} = \frac{16}{18}$$

Using the property of proportions, we obtain

$$6 \cdot 16 = 96 \quad \text{and} \quad 5 \cdot 18 = 90$$

The cross products are not the same so we do not have a proportion. ■

We use the property of proportions to find the unknown term of a proportion if three of the four terms are known.

Example 5-4 D

Find the unknown term of the given proportion. Check your solution.

$$1. \quad \frac{x}{8} = \frac{16}{64}$$

$$64 \cdot x = 8 \cdot 16$$

$$64x = 128$$

$$x = 2$$

Property of proportions

Multiply as indicated

Divide each member by 64

$$\begin{aligned} \text{Check:} \quad \frac{2}{8} &= \frac{16}{64} \quad \text{Then} \\ 2 \cdot 64 &= 8 \cdot 16 \\ 128 &= 128 \end{aligned}$$

$$2. \quad \frac{49}{y} = \frac{35}{5}$$

$$49 \cdot 5 = 35 \cdot y$$

$$35y = 245$$

$$y = 7$$

Property of proportions

Multiply as indicated

Divide each member by 35

$$\begin{aligned} \text{Check:} \quad \frac{49}{7} &= \frac{35}{5} \quad \text{Then} \\ 5 \cdot 49 &= 7 \cdot 35 \\ 245 &= 245 \end{aligned}$$

► **Quick check** Find the value of z in the proportion $\frac{72}{z} = \frac{30}{6}$. ■

Proportions are used in solving many applied problems. Consider the following examples.

■ Example 5-4 E

Set up a proportion for each problem and solve.

1. Two gears are in the ratio of 4 : 5. If the smaller gear has 32 teeth, how many teeth are there in the larger gear?

Let x = the number of teeth in the larger gear. Set up a proportion: one ratio involving 4 and the number of teeth in the smaller gear and the other involving 5 and the number of teeth in the larger gear. Corresponding numbers must be in the numerator and the denominator.

$$\begin{array}{ll} \frac{4}{32} = \frac{5}{x} & \text{Set up a proportion} \\ 4 \cdot x = 32 \cdot 5 & \text{Property of proportions} \\ 4x = 160 & \text{Multiply as indicated} \\ x = 40 & \text{Divide each member by 4} \end{array}$$

Thus, there are 40 teeth in the larger gear.

2. On a map, 1 inch represents 6 miles. How many inches are needed to represent 28 miles?

Let x = the number of inches representing 28 miles. Now, 1 inch is to 6 miles as x inches is to 28 miles.

$$\begin{array}{ll} \frac{1 \text{ in.}}{6 \text{ mi}} = \frac{x \text{ in.}}{28 \text{ mi}} & \text{Set up a proportion} \\ 6 \cdot x = 1 \cdot 28 & \text{Property of proportions} \\ x = \frac{1 \cdot 28}{6} = \frac{28}{6} & \text{Divide each member by 6} \\ = \frac{14}{3} \text{ or } 4\frac{2}{3} & \text{Reduce to lowest terms} \end{array}$$

Therefore, 28 miles are represented by $4\frac{2}{3}$ inches on the map.

Note In example 2, the same units of measure are in the numerator of the ratios and the same units of measure are in the denominators. That is, we placed inches in the numerator and miles in the denominator of each ratio. This step is important in setting up the proportion you will use to solve for the unknown.

3. Cheryl set aside \$20 per week for her savings program when her salary was \$200 per week. If her salary is now \$250 per week, how much should she set aside for her weekly savings to be proportional to what she saved before?

Let x = the amount to be set aside when Cheryl earns \$250 per week. Then, \$20 is to \$200 as x is to \$250.

$$\begin{array}{ll} \frac{20}{200} = \frac{x}{250} & \text{Set up a proportion} \\ 200 \cdot x = 20 \cdot 250 & \text{Property of proportions} \\ 200x = 5,000 & \text{Multiply as indicated} \\ x = \frac{5,000}{200} & \text{Divide each member by 200} \\ x = 25 & \end{array}$$

Cheryl should set aside \$25 when making \$250 per week.

► **Quick check** On a map, 1 inch represents 9 miles. How many inches are needed to represent 42 miles?

Mastery points

Can you

- Write ratios?
- Reduce ratios?
- Set up proportions?
- Solve proportions for the unknown?
- Set up proportions to solve problems?

Exercise 5-4

Express the given ratios in two forms, $\frac{a}{b}$ and $a : b$, reduced to lowest terms. See example 5-4 A.

Example The ratio of 18 to 21

Solution 18 : 21 or 6 : 7

Form $a : b$ (reduce by dividing each term by 3)

$$\frac{18}{21} \text{ or } \frac{6}{7}$$

Write as a fraction reduced to lowest terms

1. 12 to 7

2. 8 to 19

3. 7 to 42

4. 32 to 60

5. 16 to 6

6. 24 to 9

7. 15 to 5

8. 90 to 40

9. 8 to $\frac{3}{4}$

10. 12 to $1\frac{3}{4}$

11. $2\frac{1}{2}$ to 10

12. $5\frac{5}{6}$ to 3

13. $\frac{5}{6}$ to $\frac{2}{3}$

14. $\frac{7}{5}$ to $\frac{3}{4}$

15. $3\frac{1}{4}$ to $2\frac{2}{3}$

16. $3\frac{4}{5}$ to $4\frac{1}{10}$

17. 4.2 to 2.1

18. 2.4 to 6.0

19. 1.02 to 2.38

20. 3.06 to 2.55

Find the indicated ratios reduced to lowest terms expressed in two forms. See examples 5-4 A and B.

Example 16 minutes to 2 hours

Solution Since 2 hours = 120 minutes (1 hour = 60 minutes),

$$16 \text{ min} : 2 \text{ hr} = 16 \text{ min} : 120 \text{ min}$$

Replace 2 hr with 120 min

$$= 16 : 120$$

Eliminate unit of measure

$$= 2 : 15 \text{ or } \frac{2}{15}$$

Reduce to lowest terms (Divide by 8)

$$\text{The ratio of 16 min to 2 hr is } 2 : 15 \text{ or } \frac{2}{15}.$$

Example 64 pounds to 8 square inches

Solution 64 lb to 8 sq in. = $\frac{64 \text{ lb}}{8 \text{ sq in.}}$

Write ratio as a fraction

$$= \frac{8 \text{ lb}}{1 \text{ sq in.}}$$

Divide by 8 to reduce to lowest terms

$$= 8 \text{ lb/sq in.}$$

Write as a rate

- | | | | |
|---------------------------------|---------------------------------|-----------------------------------|----------------------|
| 21. 6 in. to 14 in. | 22. 4 ft to 18 ft | 23. 25 cm to 10 cm | 24. 35 lb to 5 lb |
| 25. 36 km to 24 km | 26. 48 lb to 16 lb | 27. 15 in. to 3 ft | 28. 10 ft to 4 yd |
| 29. \$3 to 35¢ | 30. 5 days to 15 weeks | 31. 30 min to 13 hr | 32. 16 lb to 8 oz |
| 33. 48 lb to 24 ft ³ | 34. 50 cm to 5 in. ³ | 35. 16 grams to 2 cm ³ | 36. 300 mi to 10 gal |
| 37. 1,020 mi to 17 hr | 38. 105 kg to 35 m ³ | | |

Solve the following applied problems.

39. The *output* in horsepower is the useful energy delivered by an engine and the *input* in horsepower is the amount of energy delivered to an engine. The *mechanical efficiency* of the engine is given by the ratio

$$\text{mechanical efficiency} = \frac{\text{output}}{\text{input}}$$

Find the mechanical efficiency of an engine rated to deliver 425 horsepower (input) when it delivers only 375 horsepower.

40. The *pitch* of a roof is the ratio of the *rise* of a rafter to the *span* of the roof.

$$\text{pitch} = \frac{\text{rise of rafter}}{\text{span of roof}}$$

Find the pitch if the roof rises 7 feet in a span of 21 feet.

41. The smaller of two belted pulleys makes 240 revolutions per minute and the larger one makes 100 revolutions per minute. What is the ratio of the speed of the larger pulley to the smaller pulley? Of the smaller pulley to the larger pulley?
42. The *mechanical advantage* (*MA*) of a machine is given by the ratio

$$\text{mechanical advantage (MA)} = \frac{\text{resistance to effort (output)}}{\text{input effort}}$$

If a machine has an effort of 30 pounds that results in a resistance of 120 pounds, what is the mechanical advantage of the machine?

43. The mechanical advantage of a hydraulic press can be given by the ratio

$$\text{mechanical advantage} = \frac{\text{area of the large piston}}{\text{area of the small piston}}$$

If the large piston has area 32 square centimeters and the small piston has area 12 square centimeters, find the mechanical advantage of the press.

44. Tool steel may be worked at a cutting speed of 20 feet per minute in a lathe, and cast iron may be worked at a cutting speed of 45 feet per minute. What is the ratio of the cutting speed of tool steel to cast iron?
45. An automobile engine is rated at 350 horsepower. When the engine is tested, it produces only 325 horsepower. What is the mechanical efficiency of the engine? (Refer to exercise 39.)
46. An electric motor uses 10 volts of electricity to produce an equivalent output of 8 volts. What is the mechanical efficiency of the motor? (Refer to exercise 39.)
47. A particular stock costing \$63 paid an earnings of \$6. What is the cost : earnings ratio?
48. A room is 24 feet long and 18 feet wide. What is the ratio of its length to its width?
49. A cement block weighs 120 pounds and a steel block weighs 1,860 pounds. What is the ratio of the weight of the steel block to the weight of the cement block?
50. A mathematics class contains 32 male students and 10 female students. What is the ratio of the male students to the female students?
51. A doctor having earnings of \$60,000 in a given year paid income taxes of \$8,400. What is the ratio of taxes to income?
52. The stress caused by a heavy load is defined by the ratio
- $$\text{stress} = \frac{\text{distorting force (F)}}{\text{area (A)}}$$
- measured in lb/in.² Find the stress of a force of a 4,400-pound load on an area of 1,200 square inches.
53. Power is defined by the ratio of the work done (*F*) to the time taken (*t*). Find the power if 42 ft-lb of work is done in 6 seconds, in ft-lb/sec.

54. The magnification (M) of an object by a lens is given by the ratio

$$M = \frac{q}{p}$$

where q = the image distance and p = the object distance from the lens. Find the magnification of an object whose image distance is 27 feet and object distance is 12 feet.

Find the value of the unknown that makes the proportion true. See example 5-4 D.

Example Find the value of z in the proportion $\frac{72}{z} = \frac{30}{6}$

Solution

$$\begin{aligned} \frac{72}{z} &= \frac{30}{6} \\ 30 \cdot z &= 72 \cdot 6 && \text{Property of proportions} \\ 30z &= 432 && \text{Perform indicated multiplication} \\ z &= \frac{432}{30} && \text{Divide by 30} \\ &= \frac{72}{5} \text{ or } 14\frac{2}{5} && \text{Reduce to lowest terms} \end{aligned}$$

Thus, $z = 14\frac{2}{5}$ makes the equation a proportion.

55. $\frac{9}{x} = \frac{36}{5}$

56. $\frac{y}{7} = \frac{30}{42}$

57. $\frac{5}{9} = \frac{p}{20}$

58. $\frac{14}{10} = \frac{21}{z}$

59. $6 : 15 = x : 8$

60. $R : 12 = 15 : 100$

61. $1\frac{1}{2} : a = 4\frac{3}{4} : 2$

62. $\frac{3}{4} : 4 = \frac{1}{2} : b$

63. $1.2 : x = 3.6 : 9$

64. $4.5 : 3 = y : 2$

65. $3\frac{1}{4} : \frac{5}{5} = \frac{2\frac{1}{2}}{a}$

66. $\frac{2.4}{4.2} = \frac{b}{2.1}$

Solve the following problems by first choosing a letter to represent the unknown and then setting up the proper proportion. See example 5-4 E.

Example On a map, 1 inch represents 9 miles. How many inches are needed to represent 42 miles?

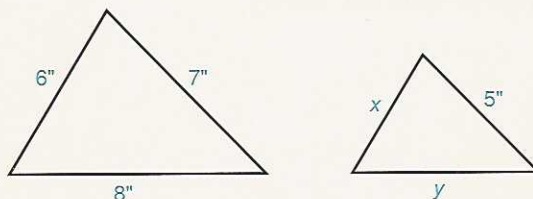
Solution Let x = the number of inches representing 42 miles.
Then, 1 inch is to 9 miles as x inches is to 42 miles.

$$\begin{aligned} \frac{1 \text{ inch}}{9 \text{ miles}} &= \frac{x \text{ inches}}{42 \text{ miles}} && \text{Set up a proportion} \\ 9 \cdot x &= 1 \cdot 42 && \text{Property of proportions} \\ 9x &= 42 && \text{Multiply as indicated} \\ x &= \frac{42}{9} && \text{Divide each member by 9} \\ &= \frac{14}{3} \text{ or } 4\frac{2}{3} && \text{Reduce to lowest terms} \end{aligned}$$

Thus, $4\frac{2}{3}$ inches represents 42 miles on the map.

67. A man earns \$180 per week. How many weeks must he work to earn \$1,260?
68. An automobile uses 8 liters of gasoline to travel 84 kilometers. How many liters are needed to travel 1,428 kilometers?
69. If 24 grams of water will yield 4 grams of hydrogen, how many grams of hydrogen will there be in 216 grams of water?
70. The operating instructions for a gasoline chain saw call for a 16 gallons : 1 pint fuel-to-oil mixture. How many *pints* of oil are needed to mix with 88 gallons of fuel?
71. The power-to-weight ratio of a given engine is 5 : 3. What is the weight of the engine if it produces 650 horsepower?
72. If a 20-pound casting costs \$1.50, at this same rate, how much would a 42-pound casting cost?
73. A copper wire 300 feet long has a resistance of 1,024 ohms. What is the resistance of 2,000 feet of copper wire?
74. In a hydraulic press, the force on the output piston is to the force on the input piston as the area of the output piston is to the area of the input piston. That is, $\frac{F_o}{F_i} = \frac{A_o}{A_i}$ or $F_o : F_i = A_o : A_i$. Find the area of the input piston if F_o is 15.2 pounds, F_i is 6.5 pounds, and A_o is 10.4 inches².
75. If the ratio of the wins to the losses of the Chicago Cubs in a given season is 6:5, how many games did they lose if they won 90 games?
76. A rectangular picture that is 10 inches long and 8 inches wide is to be enlarged so that the enlargement will be 36 inches wide. What should be the length of the enlargement?

77. The corresponding sides of the triangles in the diagram are in proportion. Find the dimensions of the missing sides, x and y . (*Hint:* The corresponding sides are 6" and x , 7" and 5", 8" and y .)



78. Ann is operating a machine that can produce 14 parts in 20 minutes. How long will it take for her to produce 224 parts?
79. A punch machine can make 72 holes in 4 minutes. How many holes can the machine make in 3 hours?
80. A roof rises $6\frac{1}{2}$ feet in a rafter span of 9 feet. At this rate, what would be the rise in a 15-foot span?
81. Nat can type 3 pages of an English paper in 15 minutes. How long would it take him to type 54 pages? (State the answer in hours and minutes.)
82. On a draftsman scale, $\frac{1}{8}$ inch represents 1 foot. What length will a measurement of $2\frac{5}{8}$ inches on the scale represent?
83. An automobile engine uses $\frac{3}{4}$ quart of oil in 900 miles. How much oil will it take in 3,000 miles?

Review exercises

Solve the following equations. See sections 2-4 and 2-7.

- $6y + 5 = y - 4$ (Find the solution set.)
- The product of two consecutive odd integers is 143. Find the integers. See section 4-8.

Completely factor each expression. See sections 4-2 and 4-4.

- $16x^2 - y^2$
- $x^2 - 16x - 17$
- $5x^2 - 5x - 10$

Student Loans for up to **\$40,000** per year*

Defer payments until after graduation.**
Fast preliminary approval, usually in minutes.

 **Apply Now!**

Go to gmacbankfunding.com

Apply online in as little as 15 minutes

- Loans up to \$40,000 per academic year*
- Good for tuition and other educational expenses: books, fees, a laptop, room and board, travel home, etc.
- Get a check in as few as 5 business days
- Start payments now or up to six months after graduation**
- Reduce your interest rate by as much as 0.50% with automatic payments***

All loans are subject to application and credit approval.

* Undergraduate and graduate borrowers may borrow annually up to the lesser of the cost of attendance or \$30,000 (\$40,000 for certain schools where it has been determined that the annual cost of attendance exceeds \$30,000). Borrowers in the Continuing Education loan program may borrow annually up to \$30,000.

** Undergraduate students may choose to defer repayment until six months after graduation or ceasing to be enrolled at least half time in school. Interest only and immediate repayment options also available.

*** A 0.25% interest rate reduction is available for borrowers who elect to have monthly principal and interest payments transferred electronically from a savings or checking account. The interest rate reduction will begin when automatic principal and interest payments start, and will remain in effect as long as automatic payments continue without interruption. This reduced interest rate will return to contract rate if automatic payments are cancelled, rejected or returned for any reason. Upon request, borrowers are also entitled to an additional 0.25% interest rate reduction if (1) the first 36 payments of principal and interest are paid on time, and (2) at any time prior to the 36th on time payment, the borrower who receives the monthly bill elects to have monthly principal and interest payments transferred electronically from a savings or checking account, and continues to make such automatic payments through the 36th payment. This reduced interest rate will not be returned to contract rate if, after receiving the benefit, the borrower discontinues automatic electronic payment. The lender and servicer reserve the right to modify or discontinue borrower benefit programs (other than the co-signer release benefit) at any time without notice.

GMAC Bank Member FDIC

Chapter 5 lead-in problem

On a road map, 3 inches represent a distance of 45 miles. If the distance between Detroit and Sault Ste. Marie is 23 inches on the map, how far is it from Detroit to Sault Ste. Marie?

Solution

Let x = the distance from Detroit to Sault Ste. Marie. Then, since 3 inches represents 45 miles on the map, we use the relationship

$$3 \text{ in. is to } 45 \text{ mi as } 23 \text{ in. is to } x \text{ mi}$$

which we write as the proportion

$$\begin{aligned}\frac{3}{45} &= \frac{23}{x} \\ 3 \cdot x &= 45 \cdot 23 && \text{Property of proportions} \\ x &= \frac{45 \cdot 23}{3} && \text{Divide each member by 3} \\ x &= 15 \cdot 23 && \text{Reduce by 3} \\ x &= 345 && \text{Multiply in the right member}\end{aligned}$$

The distance from Detroit to Sault Ste. Marie is 345 miles.

Chapter 5 summary

1. A **rational expression** can be written in the form $\frac{P}{Q}$, where P and Q are polynomials, $Q \neq 0$.
2. The **domain** of a rational expression in one variable is the set of all replacement values of the variable for which the rational expression is defined.
3. The **fundamental principle of rational expressions** is used to *reduce* rational expressions to *lowest terms* and to obtain equivalent rational expressions having the same denominator for addition and subtraction. It states, $\frac{PR}{QR} = \frac{P}{Q}$, where P , Q , and R are polynomials, Q and R are not equal to zero.
4. To **reduce** a rational expression to lowest terms, we divide the numerator and the denominator by any common factors.
5. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.
6. To divide a polynomial by a polynomial, the dividend and the divisor must be arranged in descending powers of the same variable with zeros inserted for missing variables.
7. A **ratio** is the comparison of two numbers by division.
8. A **proportion** is a statement of equality of two ratios.
9. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, where ad and bc are called the cross products.

Chapter 5 error analysis

1. Finding the domain of a rational expression

Example: $\frac{3x+1}{x^2+2x} = \frac{3x+1}{x(x+2)}$

Domain is all real numbers except -2 .

Correct answer: Domain is all real numbers except 0 and -2 .

What error was made? (see page 204)

2. Reducing to lowest terms by "cancelling"

Example: $\frac{x-3}{x^2-9} = \frac{\overset{1}{x}-\overset{1}{3}}{\underset{x}{x^2}-\underset{3}{9}} = \frac{1-1}{x-3} = \frac{0}{x-3} = 0$

Correct answer: $\frac{1}{x+3}$

What error was made? (see page 208)

3. Dividing a polynomial by a monomial

$$\text{Example: } \frac{y - y^2}{y} = \frac{\overset{1}{y} - y^2}{\underset{1}{y}} = 1 - y^2$$

Correct answer: $1 - y$

What error was made? (see page 212)

4. Dividing a polynomial by a polynomial

$$\text{Example: } \frac{x^2 - 3x - 4}{x + 1}$$

$$\begin{array}{r} x - 2 \\ x + 1 \overline{) x^2 - 3x - 4} \\ \underline{x^2 + x} \\ -2x - 4 \\ \underline{-2x - 2} \\ -6 \end{array} = x - 2 - \frac{6}{x + 1}$$

Correct answer: $x - 4$

What error was made? (see page 213)

5. Solving proportion problems

Example: On a map, 1 inch represents 10 miles. How many inches represent 24 miles? Let x = inches represented by 24 miles.

$$\frac{1}{10} = \frac{24}{x}$$

$$x = 240 \text{ inches}$$

Correct answer: 2.4 inches

What error was made? (see page 223)

6. Order of operations

$$\text{Example: } 10 - 12 \div 2 + 6 = 5$$

Correct answer: 10

What error was made? (see page 57)

7. Division using zero

$$\text{Example: } \frac{0}{-3} \text{ is undefined}$$

$$\text{Correct answer: } \frac{0}{-3} = 0$$

What error was made? (see page 53)

8. Combining like terms

$$\text{Example: } 8xy - 4xy = 4$$

Correct answer: $4xy$

What error was made? (see page 80)

9. Reciprocal of a number

Example: The reciprocal of 1 is -1 .

Correct answer: The reciprocal of 1 is itself.

What error was made? (see page 94)

10. Properties of exponents

$$\text{Example: } (3x)^3 = 3x^3$$

Correct answer: $27x^3$

What error was made? (see page 129)

Chapter 5 critical thinking

If m and n represent 2 integers where $n > m$, how many integers are there from m to n ?

Chapter 5 review

[5-1]

Determine the domain of the given rational expression.

1. $\frac{x+1}{x}$

2. $\frac{y-3}{y+7}$

3. $\frac{3x+1}{x-9}$

4. $\frac{2z-5}{3z+2}$

5. $\frac{x}{5x-3}$

6. $\frac{x^2-x+4}{x^2+x-12}$

7. $\frac{x^2+3x+2}{x^2-1}$

[5-2]

Reduce the following rational expressions to lowest terms. Assume no denominator is equal to zero.

8. $\frac{18ab^2}{6a^2b}$

9. $\frac{45x^2yz^3}{30xy^3z^2}$

10. $\frac{x^2-49}{x^2+14x+49}$

11. $\frac{x^2-3x-18}{x^2+x-42}$

12. $\frac{18a-6b}{15a-5b}$

13. $\frac{x^2-y^2}{y-x}$

14. $\frac{3p^2-8p+4}{5p^2-9p-2}$

15. $\frac{2R^2-32}{6R^2+22R-8}$

16. $\frac{20-9n+n^2}{8+2n-n^2}$

[5-3]

Find the indicated quotients.

17. $\frac{24x^3}{-3x}$

18. $\frac{2a^2 - 3a + 5a^3}{a}$

19. $\frac{5x^2y - 3xy^4 + x^2y^2}{xy}$

20. $\frac{8a^3b + 12a^2b^2 - 24a^3b^7}{4a^2b}$

21. $\frac{8a^2 - 2a - 3}{2a - 1}$

22. $\frac{3a^2 - 17a + 11}{a - 5}$

23. $\frac{x^2 - 49}{x + 7}$

24. $\frac{20x^3 - 19x^2 - 13x + 12}{4x - 3}$

[5-4]

Find the indicated ratios in two different forms reduced to lowest terms.

25. 15 meters to 35 meters

26. 36 pounds to 16 pounds

27. 12 inches to $2\frac{1}{2}$ feet

28. 450 miles to 15 gallons

29. In business, the current ratio compares current assets to current liabilities and represents the measure of the firm's ability to pay off the liabilities over a time period. What is the current ratio, reduced to lowest terms, if the firm's total current assets are \$4,386 and total current liabilities are \$1,762?

30. The May company has 42 sales representatives who are meeting their sales quota. If another 18 sales representatives have fallen short of their quota, what is the ratio of success to failure?

Find the value of the unknown that makes the statement a proportion.

31. $\frac{8}{x} = \frac{9}{36}$

32. $\frac{5.4}{3.6} = \frac{a}{2.4}$

33. $y : 18 = 15 : 25$

34. $\frac{5}{6} : \frac{1}{2} = \frac{2}{3} : p$

35. If a blueprint is drawn to the scale $\frac{1}{8}$ inch = 1 foot, what is the size of the corresponding part of a final product if the blueprint measurement is $4\frac{3}{8}$ inches?

36. An automobile has a 16-quart cooling system. If the ratio of antifreeze to water is 3 to 1, how much of each does the system have? (*Hint:* Let x be the amount of antifreeze. Then $16 - x$ is the amount of water.)

Chapter 5 cumulative test

Perform the indicated operations.

[1-8] 1. $-4[7(12 - 2) - 8^3 + 3]$

[1-7] 2. $\frac{(-6)(-8)}{(-2)(0)}$

[1-8] 3. -6^2

Perform the indicated operations and simplify.

[2-3] 4. $(5a - b) - [3a - (4b + 3a)]$

[3-2] 5. $(3x - 2)^2$

[3-2] 6. $(5y - 2)(5y + 2)$

[3-2] 7. $(4a + 3b)(a - 6b)$

[3-2] 8. $(x - y)^3$

[2-2] 9. Given $a = -5$, $b = 3$, $c = 4$, and $d = -6$, evaluate the expression $(2a - 3b) - (5c + d)$.

Find the solution set.

[2-6] 10. $3(x + 2) - 2(x - 4) = 12$

[2-6] 11. $\frac{3x}{4} - 5 = 1$

[2-9] 12. $-4 \leq 2x + 5 < 11$

[2-9] 13. $2(3x - 4) > 5(x - 1)$

Write in completely factored form.

[4-1] 14. $x(m + n) - y(m + n)$

[4-3] 15. $3a^2 + 7a + 4$

[4-4] 16. $4x^2 - 20x + 25$

[4-4] 17. $4z^2 - 9$

[4-4] 18. $36 - y^2$

[4-2] 19. $x^2 - 12x - 45$

[4-5] 20. $2a^3 - 16b^3$

Find the solution set.

[4-7] 21. $x^2 - 5x - 14 = 0$

[4-7] 22. $2y^2 + 3y - 9 = 0$

Simplify and leave answers with only positive exponents.

[3-4] 23. $(4yz^{-1})^{-3}$

[3-4] 24. $\frac{a^{-7}}{a^{-10}}$

[3-4] 25. $(4ab^2)(-2a^3b)(-a^{-1}b^{-3})$

[5-4] 26. Find x when $36 : x = 21 : 14$.

[5-4] 27. What is the ratio of 52 pounds to 24 pounds?

[5-3] 28. Divide $(x^2 - 8x + 13) \div (x - 2)$.

Reduce the following expressions to lowest terms. Assume that no denominator is equal to zero.

[1-1] 29. $\frac{56}{42}$

[3-3] 30. $\frac{36ab^3}{28a^3b^2}$

[5-2] 31. $\frac{a^2 - 36}{a^2 - a - 42}$

[5-2] 32. $\frac{8x - 8y}{5x^2 - 5y^2}$

[5-2] 33. $\frac{y^2 - y - 20}{y^2 - 25}$

[5-2] 34. $\frac{2x^2 - 5xy - 3y^2}{6x^2 + 7xy + 2y^2}$

[5-4] 35. A photograph that is 8 inches by 12 inches is to be enlarged. If the enlargement calls for the longest side to be 32 inches, how many inches should the other side be?

[5-4] 36. Two lots are proportional in their lengths and widths. If the larger lot is 15 feet wide and 28 feet long and the length of the smaller lot is 24 feet, how wide is the smaller lot?

16. $(3R + 1)(a + b)$ 17. $(x - 3y)(2a - 3b)$
 18. $(3a - b)(2x - y)$ 19. $(x + 2y)(4a + 3b)$
 20. $(a + 3b)(x - 4)$ 21. $(x^2 + 4)(a - 2b)$
 22. $(x - 7)(x - 2)$ 23. $2a(a - 5)(a + 1)$
 24. $(a + 12)(a + 2)$ 25. $(x - 8)(x + 4)$
 26. $(a - 18)(a + 2)$ 27. $3(x - 5)(x + 2)$
 28. $x(x - 3)(x + 2)$ 29. $x(x - 7)(x + 3)$
 30. $(ab + 3)(ab - 2)$ 31. $(ab + 6)(ab + 4)$
 32. $(ab - 6)(ab - 3)$ 33. $(ab - 10)(ab + 2)$
 34. $(2x + 1)(2x + 1)$ or $(2x + 1)^2$ 35. $9(r - 2)(r - 2)$ or $9(r - 2)^2$ 36. $(4x - 1)(x - 1)$ 37. $(3a + 5)(3a - 2)$
 38. $(4a - 3)(2a + 1)$ 39. $(6x + 1)(4x + 3)$
 40. $(4a - 3)(2a - 3)$ 41. $(2a + 3)(a + 6)$
 42. $(2a + 3)(2a - 3)$ 43. $(6b + c)(6b - c)$
 44. $(5 + a)(5 - a)$ 45. $4(2x + y)(2x - y)$
 46. $(3x + y^2)(3x - y^2)$ 47. $(x^2 + 4)(x + 2)(x - 2)$
 48. $(y^2 + 9)(y + 3)(y - 3)$ 49. $(b + 6)^2$ 50. $(c - 5)^2$
 51. $(2x - 3)^2$ 52. $(3x - 2)^2$ 53. $(R + 2S)(R^2 - 2RS + 4S^2)$
 54. $2(2x - 3)(4x^2 + 6x + 9)$ 55. $(3a + 5b)(9a^2 - 15ab + 25b^2)$
 56. $(xy - 1)(x^2y^2 + xy + 1)$ 57. $2(x^3 + 5)(x^6 - 5x^3 + 25)$
 58. $(4x^4 - y^5)(16x^8 + 4x^4y^5 + y^{10})$
 59. $(ab^2 + c^3)(a^2b^4 - ab^2c^3 + c^6)$ 60. $3x^3(4x - 1)$
 61. $(a - 5)(a + 2)$ 62. $(4a - 1)(a - 5)$
 63. $(3y + 2)(3y - 2)$ 64. $(2a + 3b)(3x - 2)$
 65. $(b - 5)(b + 4)$ 66. $(3x + 2)(3x + 5)$ 67. $(a + 7)^2$
 68. $3x^3(2x + 1)(2x - 1)$ 69. $c(c + 4)(c + 5)$ 70. $(4a - 1)^2$
 71. $(b^2 + 1)(b + 1)(b - 1)$ 72. $\{1, -3\}$ 73. $\{0, 8\}$
 74. $\left\{-\frac{1}{5}, \frac{7}{3}\right\}$ 75. $\left\{\frac{1}{7}, \frac{8}{5}\right\}$ 76. $\left\{\frac{4}{3}, 9\right\}$ 77. $\{0, -9, -4\}$
 78. $\left\{\frac{4}{5}, -\frac{4}{5}\right\}$ 79. $\{-1, 3, -2\}$ 80. $\left\{0, \frac{9}{4}\right\}$ 81. $\{-1, 1\}$
 82. $\{0, 64\}$ 83. $\{-5, 5\}$ 84. $\{6, -5\}$ 85. $\{1\}$ 86. $\left\{-\frac{1}{4}, -3\right\}$
 87. $\left\{-\frac{2}{5}, 2\right\}$ 88. $\{4\}$ 89. $\{1, 3\}$ 90. $\left\{-1, \frac{3}{4}\right\}$ 91. $\{2, -3\}$
 92. 9, 10 93. 8 feet and 13 feet 94. 20 cattle 95. 6 seconds

Chapter 4 cumulative test

1. 33 2. $8a^6b^3$ 3. $a^2 + 4ab + 4b^2$ 4. a^6 5. 12 6. x^5y^4
 7. $5x - 7y$ 8. $9x^2 - 4y^2$ 9. $\frac{8a^6}{b^3}$ 10. $\frac{1}{x^2}$ 11. $4x + 4y$
 12. $\frac{x^6}{9y^4}$ 13. a. 0 b. 0 c. -6 14. $\left\{\frac{9}{5}\right\}$ 15. $x < 12$
 16. $\{7\}$ 17. $x \geq 3$ 18. $\{-3, 3\}$ 19. $\{2, 5\}$ 20. $x < \frac{5}{2}$
 21. $2 \leq x \leq \frac{14}{3}$ 22. $x = 3y$ 23. $x = \frac{5y + 2}{3}$
 24. $2ab(1 - 2ab - 4a^2b^4)$ 25. $(2a + 3)^2$
 26. $(5c + 3d)(5c - 3d)$ 27. $(2a + 3)(2a - 5)$
 28. $(x + 3)(x + 6)$ 29. 14, 39 30. 11, 13
 31. \$10,000 at 8%; \$5,000 at 6% 32. 6 meters by 11 meters

Chapter 5

Exercise 5-1

Answers to odd-numbered problems

1. $\frac{1}{3}$ 3. undefined 5. 4 7. $\frac{13}{23}$ 9. $\frac{16}{17}$ 11. 3
 13. undefined 15. $\frac{15}{2}$ 17. all real numbers except 0
 19. all real numbers except 5 21. all real numbers except -3
 23. all real numbers except $\frac{3}{4}$ 25. all real numbers except $\frac{8}{3}$
 27. all real numbers except 1 and 6 29. all real numbers
 except -2 and $\frac{4}{3}$ 31. all real numbers except $-\frac{2}{3}$ and $\frac{2}{3}$
 33. all real numbers except -3 and 3 35. all real numbers
 except $\frac{7}{3}$ 37. all real numbers except -1 and 1 39. all real
 numbers 41. $L = 0$ 43. $T_1 \neq 0, T_2 \neq 0$

Solutions to trial exercise problems

6. $\frac{-5b^3}{5 - 2b}; b = -2$

$$\frac{-5b^3}{5 - 2b} = \frac{-5(-2)^3}{5 - 2(-2)} = \frac{-5(-8)}{5 + 4} = \frac{40}{9}$$

 9. $\frac{(-2x)^2}{x^2 + 3x + 7}; x = 2$

$$\frac{(-2x)^2}{x^2 + 3x + 7} = \frac{[(-2)(2)]^2}{(2)^2 + 3(2) + 7} = \frac{(-4)^2}{4 + 6 + 7} = \frac{16}{17}$$

 22. $\frac{x + 1}{2x - 1}$ Set $2x - 1 = 0$, then $2x = 1$ and $x = \frac{1}{2}$.
 Domain is all real numbers except $\frac{1}{2}$.
 25. $\frac{y + 4}{8 - 3y}$ Set $8 - 3y = 0$, then $3y = 8$ and $y = \frac{8}{3}$.
 Domain is all real numbers except $\frac{8}{3}$.
 28. $\frac{5s^2 + 7}{2s^2 - s - 3}$ Set $2s^2 - s - 3 = 0$ and factor. We have

$$(2s - 3)(s + 1) = 0. \text{ Then}$$

$$2s - 3 = 0 \text{ or } s + 1 = 0$$

$$2s = 3 \quad s = -1$$

$$s = \frac{3}{2} \quad s = -1$$

 Domain is all real numbers except -1 and $\frac{3}{2}$.
 32. $\frac{a - 2}{4a^2 - 16}$ Set $4a^2 - 16 = 0$ and factor. We have

$$4(a - 2)(a + 2) = 0.$$
 This is true if and only if $a - 2 = 0, a = 2$ or
 $a + 2 = 0, a = -2$.
 Domain is all real numbers except -2 and 2.
 37. $\frac{17q}{3q^2 - 3}$ Set $3q^2 - 3 = 0$ and factor to get

$$3(q^2 - 1) = 0$$

$$3(q + 1)(q - 1) = 0$$

$$q + 1 = 0 \text{ or } q - 1 = 0$$
 Then $q = -1$ or $q = 1$
 Domain is all real numbers except -1 and 1.

Review exercises

1. distributive property 2. -50 3. 5 4. $\frac{5}{9}$
 5. $(2x + 1)(x - 5)$ 6. $4(y - 5)^2$

Exercise 5-2

Answers to odd-numbered problems

1. $\frac{3}{4}$ 3. $\frac{2x}{5}$ 5. $\frac{4x}{3}$ 7. $-\frac{4}{3x^2}$ 9. $\frac{4a}{5b}$ 11. -5 13. $\frac{5}{4}$
 15. $\frac{6}{x+3}$ 17. $\frac{1}{a-b}$ 19. $\frac{3}{5}$ 21. $\frac{x-1}{2(x+1)}$ 23. $\frac{x-3}{x+3}$
 25. $\frac{x-5}{x-3}$ 27. $\frac{2y+3}{4y-1}$ 29. $\frac{1}{x^2+3x+9}$ 31. $\frac{a-b}{a^2-ab+b^2}$
 33. -4 35. $-\frac{2}{3}$ 37. $-2(x+y)$ 39. $\frac{y-x}{x+y}$ 41. $\frac{n-m}{n+m}$
 43. $-\frac{4}{x+y}$ 45. $-\frac{1}{x+4}$

Solutions to trial exercise problems

10. $\frac{15a^2x^3}{35ax^2} = \frac{3 \cdot 5 \cdot a \cdot a \cdot x \cdot x \cdot x}{7 \cdot 5 \cdot a \cdot x \cdot x} = \frac{3 \cdot a \cdot x \cdot (5 \cdot a \cdot x \cdot x)}{7 \cdot (5 \cdot a \cdot x \cdot x)} = \frac{3ax}{7}$
 17. $\frac{a+b}{a^2-b^2} = \frac{a+b}{(a+b)(a-b)} = \frac{1}{a-b}$
 25. $\frac{x^2-3x-10}{x^2-x-6} = \frac{(x-5)(x+2)}{(x-3)(x+2)} = \frac{x-5}{x-3}$
 30. $\frac{x+2}{x^3+8} = \frac{x+2}{(x+2)(x^2-2x+4)} = \frac{1}{x^2-2x+4}$
 34. $\frac{8b-8a}{a-b} = \frac{-8(a-b)}{a-b} = -8$
 45. $\frac{x-3}{12-x-x^2} = \frac{x-3}{(3-x)(4+x)} = \frac{-1(3-x)}{(3-x)(4+x)} = \frac{-1}{x+4}$

Review exercises

1. 3.14×10^{-4} 2. $-\frac{11}{7}$ 3. 5 ft and 11 ft 4. 1 5. $27y^3$
 6. $-8x^3y^6$

Exercise 5-3

Answers to odd-numbered problems

1. $4x^2$ 3. $-5x^3yz$ 5. $3(a-b)$ 7. $2a(b-c)$
 9. $-4a^2b(x+y)$ 11. $-4a^2+2a$ 13. a^2-3a+2
 15. $5a^2-3a+4-\frac{2}{a}$ 17. $-x+y+2y^2$
 19. $10xy^2+7-6y^2$ 21. $a-c$ 23. $a+9+\frac{28}{a-2}$
 25. $a+2+\frac{4}{a+3}$ 27. $a+3+\frac{1}{a+3}$
 29. $3a-4-\frac{4}{3a-4}$ 31. x^2+2x+4 33. x^2+2x+3
 35. $b^2+7b+14+\frac{6}{b-1}$ 37. $3a+8$
 39. $x^2-1+\frac{6x-13}{x^2+3x-5}$ 41. $y^2-2y+9+\frac{-26y+42}{y^2+2y-5}$
 43. $4x+3$ 45. $-6x^4+15x^3+4x^2-22x+30$

Solutions to trial exercise problems

5. $\frac{3(a-b)^2}{a-b} = \frac{3(a-b)^2}{(a-b)} = 3(a-b)^{2-1} = 3(a-b)^1 = 3(a-b)$
 or $3a-3b$ (Note: A quantity is treated as just one term.)
 15. $\frac{15a^3-9a^2+12a-6}{3a} = \frac{15a^3}{3a} - \frac{9a^2}{3a} + \frac{12a}{3a} - \frac{6}{3a}$
 $= 5a^2 - 3a + 4 - \frac{2}{a}$
 17. $\frac{x^2y-xy^2-2xy^3}{-xy} = \frac{x^2y}{-xy} - \frac{xy^2}{-xy} - \frac{2xy^3}{-xy} = -x + y + 2y^2$
 21. $\frac{a(b-1)-c(b-1)}{b-1} = \frac{a(b-1)}{(b-1)} - \frac{c(b-1)}{(b-1)} = a - c$
 30. $(27a^3-1) \div (3a-1)$
 (Note: Insert zeros to hold positions where terms are missing.)

$$\begin{array}{r} 9a^2+3a+1 \\ 3a-1 \overline{) 27a^3+0a^2+0a-1} \\ \underline{27a^3-9a^2} \\ 9a^2+0a \\ \underline{9a^2-3a} \\ 3a-1 \\ \underline{3a-1} \\ 0 \end{array}$$

42. The length times the width is x^2+6x+8 . If we know the length to be $x+4$, then the width is found by

$$\begin{array}{r} x+2 \\ x+4 \overline{) x^2+6x+8} \\ \underline{x^2+4x} \\ 2x+8 \\ \underline{2x+8} \\ 0 \end{array} \quad \text{Width} = x+2$$

Review exercises

1. $\left\{-2, -\frac{1}{4}\right\}$ 2. 6 ft and 7 ft 3. $16x^2-24x+9$
 4. x^3+3x^2+x-2 5. $25y^2-1$ 6. $\frac{3y+1}{2y+3}$

Exercise 5-4

Answers to odd-numbered problems

1. $\frac{12}{7}$; 12:7 3. $\frac{1}{6}$; 1:6 5. $\frac{8}{3}$; 8:3 7. $\frac{3}{1}$; 3:1
 9. $\frac{32}{3}$; 32:3 11. $\frac{1}{4}$; 1:4 13. $\frac{5}{4}$; 5:4 15. $\frac{39}{32}$; 39:32
 17. $\frac{2}{1}$; 2:1 19. $\frac{3}{7}$; 3:7 21. $\frac{3}{7}$; 3:7 23. $\frac{5}{2}$; 5:2
 25. $\frac{3}{2}$; 3:2 27. $\frac{5}{12}$; 5:12 29. $\frac{60}{7}$; 60:7 31. $\frac{1}{26}$; 1:26
 33. $\frac{2 \text{ lb}}{1 \text{ ft}^3} = 2 \text{ lb:1 ft}^3$ 35. $\frac{8}{1} \left(\frac{\text{g}}{\text{cm}^3}\right)$; 8 g:1 cm³
 37. $\frac{60}{1} \left(\frac{\text{miles}}{\text{hr}}\right)$; 60 miles:1 hr 39. $\frac{15}{17}$ 41. $\frac{5}{12}$; $\frac{5}{12}$
 43. $\frac{8}{3}$ 45. $\frac{13}{14}$ 47. $\frac{21}{2}$ 49. $\frac{31}{2}$ 51. $\frac{7}{50}$
 53. 7 ft-lb/sec 55. $x = \frac{5}{4}$ 57. $p = \frac{100}{9}$ 59. $x = \frac{16}{5}$
 61. $a = \frac{12}{19}$ 63. $x = 3$ 65. $a = \frac{50}{13}$ 67. 7 weeks

69. 36 grams of hydrogen 71. 390 73. $6,826\frac{2}{3}$ ohms
 75. 75 losses 77. $x = \frac{30}{7}$ inches, $y = \frac{40}{7}$ inches
 79. 3,240 holes 81. 270 minutes = $4\frac{1}{2}$ hours 83. $2\frac{1}{2}$ quarts

Solutions to trial exercise problems

9. 8 to $\frac{3}{4}$ is written $\frac{8}{\frac{3}{4}}$ or $8:\frac{3}{4}$

but $\frac{8}{\frac{3}{4}} = 8 \cdot \frac{4}{3} = \frac{32}{3}$

Therefore we have the ratio $\frac{32}{3}$ or 32:3

13. $\frac{5}{6}$ to $\frac{2}{3}$ is written $\frac{\frac{5}{6}}{\frac{2}{3}} = \frac{5}{6} \cdot \frac{3}{2} = \frac{5}{4}$

Therefore we have the ratio $\frac{5}{4}$ or 5:4

17. 4.2 to 2.1 is written $\frac{4.2}{2.1} = \frac{42}{21} = \frac{2}{1}$

so we have the ratio $\frac{2}{1}$ or 2:1

27. Since there are $3 \cdot 12 = 36$ inches in 3 feet, we have

15 inches to 36 inches written $\frac{15}{36} = \frac{5}{12}$ or 5:12

36. 300 miles to 10 gallons is written $\frac{300 \text{ miles}}{10 \text{ gallons}} = \frac{30 \text{ miles}}{1 \text{ gallon}}$,

which we state as 30 miles per gallon.

39. $ME = \frac{\text{output}}{\text{input}} = \frac{375}{425} = \frac{15}{17}$

Therefore the mechanical efficiency is $\frac{15}{17}$

44. $\frac{\text{Cutting speed of tool steel}}{\text{Cutting speed of cast iron}} = \frac{20 \text{ ft/min}}{45 \text{ ft/min}} = \frac{4}{9}$

53. $\frac{F}{t} = \frac{42 \text{ ft-lb}}{6 \text{ sec}} = 7 \text{ ft-lb per sec}$

55. $\frac{9}{x} = \frac{36}{5}$, then $36 \cdot x = 9 \cdot 5$

$36x = 45$

$x = \frac{45}{36} = \frac{5}{4}$

Therefore $\frac{9}{\frac{5}{4}} = \frac{36}{5}$

59. 6:15 = x:8, then $15 \cdot x = 6 \cdot 8$

$15x = 48$

$x = \frac{48}{15} = \frac{16}{5}$

Therefore 6:15 = $\frac{16}{5}$:8

62. $\frac{3}{4} : 4 = \frac{1}{2} : b$, then $\frac{3}{4} \cdot b = 4 \cdot \frac{1}{2}$

$\frac{3}{4}b = 2$

$b = 2 \cdot \frac{4}{3} = \frac{8}{3}$

so $\frac{3}{4} : 4 = \frac{1}{2} : \frac{8}{3}$

68. Let x = the number of liters of gasoline to travel 1,428 kilometers.

Then $\frac{8 \text{ liters}}{84 \text{ km}} = \frac{x \text{ liters}}{1,428 \text{ km}}$ and

$x \cdot 84 = 8 \cdot 1,428$

$84x = 11,424$

$x = \frac{11,424}{84} = 136$

Therefore at the same rate of gasoline consumption, it would take 136 liters to travel 1,428 kilometers.

76. Let ℓ = the length of the enlargement.

Then $\frac{10 \text{ in.}}{8 \text{ in.}} = \frac{\ell \text{ in.}}{36 \text{ in.}}$ and $8 \cdot \ell = 10 \cdot 36$

$8\ell = 360$

$\ell = \frac{360}{8} = 45$

Therefore the enlargement will be 45 inches long.

82. Let x = the number of feet represented by $2\frac{5}{8}$ inches.

Then $\frac{\frac{1}{8} \text{ in.}}{1 \text{ ft}} = \frac{2\frac{5}{8} \text{ in.}}{x \text{ ft}}$ and $\frac{1}{8} \cdot x = 1 \cdot 2\frac{5}{8}$

$\frac{1}{8}x = 2\frac{5}{8}$

$x = \frac{21}{8} \cdot 8 = 21$

Therefore $2\frac{5}{8}$ inches represents 21 feet.

Review exercises

1. $\left\{-\frac{9}{5}\right\}$ 2. $w = \frac{P-2\ell}{2}$ 3. 11 and 13; -13 and -11

4. $x \leq 4$ 5. $(4x+y)(4x-y)$ 6. $(x-17)(x+1)$

7. $5(x-2)(x+1)$

Chapter 5 review

1. All real numbers except 0 2. All real numbers except -7

3. All real numbers except 9 4. All real numbers except $-\frac{2}{3}$

5. All real numbers except $\frac{3}{5}$ 6. All real numbers except -4

and 3 7. All real numbers except -1 and 1 8. $\frac{3b}{a}$ 9. $\frac{3xz}{2y^2}$

10. $\frac{x-7}{x+7}$ 11. $\frac{x+3}{x+7}$ 12. $\frac{6}{5}$ 13. $-(x+y)$ 14. $\frac{3p-2}{5p+1}$

15. $\frac{R-4}{3R-1}$ 16. $\frac{5-n}{2+n}$ 17. $-8x^2$ 18. $2a-3+5a^2$
 19. $5x-3y^3+xy$ 20. $2a+3b-6ab^6$
 21. $4a+1+\frac{-2}{2a-1}$ 22. $3a-2+\frac{1}{a-5}$ 23. $x-7$
 24. $5x^2-x-4$ 25. $\frac{3}{7}$ or 3:7 26. $\frac{9}{4}$ or 9:4 27. $\frac{2}{5}$ or 2:5
 28. 30 $\frac{\text{miles}}{\text{gallon}}$ or 30 miles per gallon 29. $\frac{2,193}{881}$ or 2,193:881
 30. $\frac{7}{3}$ or 7:3 31. $x=32$ 32. $a=3.6$ 33. $y=\frac{54}{5}$
 34. $p=\frac{2}{5}$ 35. 35 feet 36. 12 quarts antifreeze, 4 quarts water

Chapter 5 cumulative test

1. 1,756 2. undefined 3. -36 4. $5a+3b$
 5. $9x^2-12x+4$ 6. $25y^2-4$ 7. $4a^2-21ab-18b^2$
 8. $x^3-3x^2y+3xy^2-y^3$ 9. -33 10. $\{-2\}$ 11. $\{8\}$
 12. $-\frac{9}{2} \leq x < 3$ 13. $x > 3$ 14. $(m+n)(x-y)$
 15. $(3a+4)(a+1)$ 16. $(2x-5)^2$ 17. $(2z+3)(2z-3)$
 18. $(6-y)(6+y)$ 19. $(x-15)(x+3)$
 20. $2(a-2b)(a^2+2ab+4b^2)$ 21. $\{7, -2\}$ 22. $\left\{\frac{3}{2}, -3\right\}$
 23. $\frac{z^3}{64y^3}$ 24. a^3 25. $8a^3$ 26. $x=24$ 27. $\frac{13}{6}$ or 13:6
 28. $x-6+\frac{1}{x-2}$ 29. $\frac{4}{3}$ 30. $\frac{9b}{7a^2}$ 31. $\frac{a-6}{a-7}$
 32. $\frac{8}{5(x+y)}$ 33. $\frac{y+4}{y+5}$ 34. $\frac{x-3y}{3x+2y}$ 35. $21\frac{1}{3}$ inches
 36. $12\frac{6}{7}$ feet

Chapter 6

Exercise 6-1

Answers to odd-numbered problems

1. $\frac{3}{5}$ 3. $\frac{5}{6}$ 5. $2a$ 7. 10 9. $\frac{x}{4y}$ 11. $\frac{3x}{4}$ 13. $\frac{4}{35x}$
 15. $\frac{35x}{4}$ 17. $\frac{7c}{2b}$ 19. $\frac{x}{6y^2}$ 21. $\frac{16bcx}{3az}$ 23. $\frac{2a}{5b^2x}$
 25. $\frac{4}{x+y}$ 27. $-\frac{3}{4}$ 29. $-\frac{15}{4}$ 31. $\frac{4}{a-5}$ 33. $\frac{18(x-2)}{x+2}$
 35. $\frac{24y(x-2)}{25}$ 37. $(r-4)(r-1)$ 39. $-4(x+3)$
 41. $\frac{a-3}{a-5}$ 43. $\frac{(x-3)(x+1)}{(x-1)(x+2)}$ 45. $\frac{(2x-1)(x-1)}{(x-8)(x+7)}$
 47. 1 49. $3x+4$ 51. $\frac{1}{(2x+1)^2}$ 53. $\frac{2a+12}{3a^2+9a+27}$
 55. $\frac{(z-7)(z+3)}{5(z^2-2z+4)}$ 57. $y+5$

Solutions to trial exercise problems

10. $\frac{7a}{12b} \cdot \frac{9b}{28} = \frac{7a \cdot 9b}{12b \cdot 28} = \frac{7 \cdot a \cdot 3 \cdot 3 \cdot b}{2 \cdot 2 \cdot 3 \cdot b \cdot 2 \cdot 2 \cdot 7}$
 $= \frac{3 \cdot a \cdot (7 \cdot 3 \cdot b)}{2 \cdot 2 \cdot 2 \cdot 2 \cdot (7 \cdot 3 \cdot b)} = \frac{3a}{16}$
 20. $\frac{28m}{15n} \div \frac{7m^2}{3n^3} = \frac{28m \cdot 3n^3}{15n \cdot 7m^2} = \frac{4n^2}{5m}$
 21. $\frac{24abc}{7xyz^2} \cdot \frac{14x^2yz}{9a^2} = \frac{3 \cdot 8 \cdot a \cdot b \cdot c \cdot 2 \cdot 7 \cdot x^2 \cdot y \cdot z}{7 \cdot x \cdot y \cdot z^2 \cdot 3 \cdot 3 \cdot a^2}$
 $= \frac{8 \cdot b \cdot c \cdot 2 \cdot x(3 \cdot 7 \cdot a \cdot x \cdot y \cdot z)}{z \cdot 3 \cdot a(3 \cdot 7 \cdot a \cdot x \cdot y \cdot z)} = \frac{8 \cdot b \cdot c \cdot 2 \cdot x}{z \cdot 3 \cdot a} = \frac{16bcx}{3az}$
 25. $\frac{x+y}{3} \cdot \frac{12}{(x+y)^2} = \frac{2 \cdot 2 \cdot 3(x+y)}{3(x+y)^2} = \frac{4}{x+y}$
 30. $\frac{8y+16}{3-y} \cdot \frac{4y-12}{3y+6} = \frac{8(y+2) \cdot 4(y-3)}{-(y-3) \cdot 3(y+2)} = \frac{8 \cdot 4}{-3} = -\frac{32}{3}$
 37. $\frac{r^2-16}{r+1} \div \frac{r+4}{r^2-1} = \frac{(r^2-16)(r^2-1)}{(r+1)(r+4)}$
 $= \frac{(r-4)(r+4)(r+1)(r-1)}{(r+1)(r+4)} = \frac{(r-4)(r-1)}{1}$
 $= r^2 - 5r + 4$
 39. $\frac{9-x^2}{x+y} \cdot \frac{4x+4y}{x-3} = \frac{(3-x)(3+x) \cdot 4(x+y)}{(x+y)(x-3)}$
 $= \frac{-1(x-3)(x+3) \cdot 4(x+y)}{(x+y)(x-3)} = \frac{-1(x+3) \cdot 4}{1}$
 $= -4(x+3) = -4x - 12$
 41. $\frac{a^2-5a+6}{a^2-9a+20} \cdot \frac{a^2-5a+4}{a^2-3a+2}$
 $= \frac{(a-3)(a-2) \cdot (a-4)(a-1)}{(a-4)(a-5) \cdot (a-2)(a-1)} = \frac{a-3}{a-5}$
 47. $\frac{6r^2-r-7}{12r^2+16r-35} \div \frac{r^2-r-2}{2r^2+r-10}$
 $= \frac{(6r^2-r-7)(2r^2+r-10)}{(12r^2+16r-35)(r^2-r-2)}$
 $= \frac{(6r-7)(r+1)(2r+5)(r-2)}{(6r-7)(2r+5)(r-2)(r+1)} = 1$
 49. $(3x^2-2x-8) \div \frac{x^2-4}{x+2} = \frac{(3x+4)(x-2)}{1}$

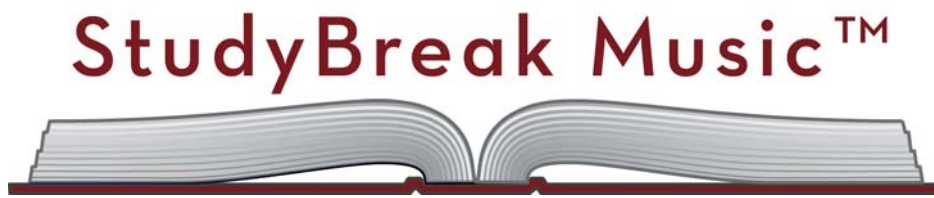
$$\cdot \frac{x+2}{(x+2)(x-2)} = 3x+4$$

$$53. \frac{10}{a^3-27} \cdot \frac{a^2+3a-18}{15} = \frac{2 \cdot 5(a+6)(a-3)}{3 \cdot 5(a-3)(a^2+3a+9)}$$

$$= \frac{2(a+6)}{3(a^2+3a+9)} = \frac{2a+12}{3a^2+9a+27}$$

Review exercises

1. $\frac{19}{12}$ 2. $\frac{11}{24}$ 3. $2(x+5)(x-5)$ 4. $(x+11)(x-2)$
 5. $(x+4)^2$ 6. $x=\frac{24}{5}$ 7. $y=15$ 8. 7.89×10^{-5}



Free tunes.
For Students. By Students.

Available for Fall 07 StudyBreaks
www.freeloadpress.com

Student musicians: Share your music. Make some change.

Learn more: www.freeloadpress.com/studybreakmusic

Contents

20 Point Learning System	xi
Preface	xvii
Study Tips	xxiii

Chapter 1 ■ Operations with real numbers



1-1 Operations with fractions	2
1-2 Operations with decimals and percents	14
1-3 The set of real numbers and the real number line	25
1-4 Addition of real numbers	34
1-5 Subtraction of real numbers	41
1-6 Multiplication of real numbers	46
1-7 Division of real numbers	51
1-8 Properties of real numbers and order of operations	56
Chapter 1 lead-in problem	62
Chapter 1 summary	63
Chapter 1 error analysis	64
Chapter 1 critical thinking	64
Chapter 1 review	64

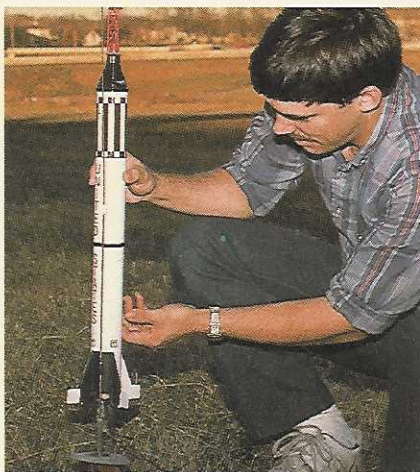
Chapter 2 ■ Solving equations and inequalities



2-1 Algebraic notation and terminology	67
2-2 Evaluating algebraic expressions	72
2-3 Algebraic addition and subtraction	79
2-4 The addition and subtraction property of equality	86
2-5 The multiplication and division property of equality	93
2-6 Solving linear equations	98
2-7 Solving literal equations and formulas	104
2-8 Word problems	107
2-9 Solving linear inequalities	113
Chapter 2 lead-in problem	122
Chapter 2 summary	122
Chapter 2 error analysis	123
Chapter 2 critical thinking	123
Chapter 2 review	124
Chapter 2 cumulative test	125

Chapter 3 ■ Polynomials and exponents

3-1	Exponents—I	127
3-2	Products of algebraic expressions	133
3-3	Exponents—II	139
3-4	Exponents—III	145
3-5	Scientific notation	148
	Chapter 3 lead-in problem	151
	Chapter 3 summary	152
	Chapter 3 error analysis	152
	Chapter 3 critical thinking	152
	Chapter 3 review	153
	Chapter 3 cumulative test	154

Chapter 4 ■ Factoring and solution of quadratic equations by factoring

4-1	Common factors	155
4-2	Factoring trinomials of the form $x^2 + bx + c$	162
4-3	Factoring trinomials of the form $ax^2 + bx + c$	166
4-4	Factoring the difference of two squares and perfect square trinomials	175
4-5	Other types of factoring	179
4-6	Factoring: A general strategy	184
4-7	Solving quadratic equations by factoring	186
4-8	Applications of the quadratic equation	193
	Chapter 4 lead-in problem	197
	Chapter 4 summary	198
	Chapter 4 error analysis	198
	Chapter 4 critical thinking	198
	Chapter 4 review	199
	Chapter 4 cumulative test	200

Chapter 5 ■ Rational Expressions, Ratio and Proportion

5-1	Rational numbers and rational expressions	202
5-2	Simplifying rational expressions	207
5-3	The quotient of two polynomials	212
5-4	Ratio and proportion	219
	Chapter 5 lead-in problem	228
	Chapter 5 summary	228
	Chapter 5 error analysis	228
	Chapter 5 critical thinking	229
	Chapter 5 review	229
	Chapter 5 cumulative test	230

Chapter 6 ■ Operations with Rational Expressions



6-1	Multiplication and division of rational expressions	232
6-2	Addition and subtraction of rational expressions	239
6-3	Addition and subtraction of rational expressions	245
6-4	Complex fractions	253
6-5	Rational equations	258
6-6	Rational expression applications	265
	Chapter 6 lead-in problem	270
	Chapter 6 summary	271
	Chapter 6 error analysis	271
	Chapter 6 critical thinking	272
	Chapter 6 review	272
	Chapter 6 cumulative test	274

Chapter 7 ■ Linear Equations in Two Variables



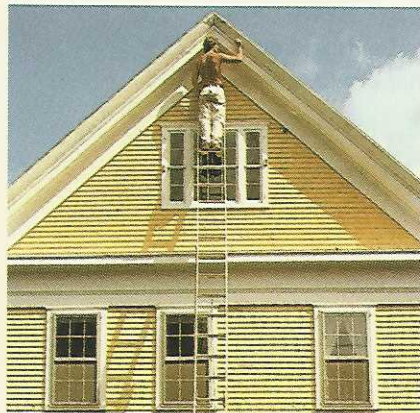
7-1	Ordered pairs and the rectangular coordinate system	276
7-2	Graphs of linear equations	289
7-3	The slope of a line	297
7-4	The equation of a line	305
7-5	Graphing linear inequalities in two variables	315
7-6	Functions defined by linear equations in two variables	323
	Chapter 7 lead-in problem	329
	Chapter 7 summary	329
	Chapter 7 error analysis	330
	Chapter 7 critical thinking	330
	Chapter 7 review	331
	Chapter 7 cumulative test	333

Chapter 8 ■ Systems of Linear Equations



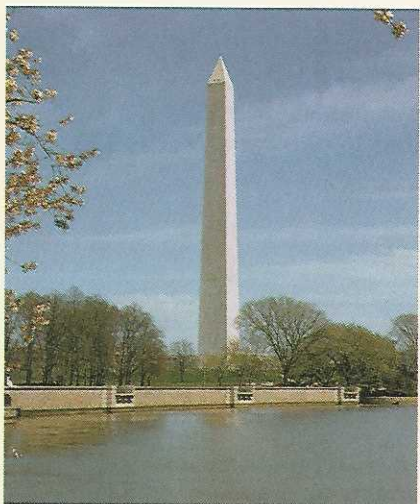
8-1	Solutions of systems of linear equations by graphing	335
8-2	Solutions of systems of linear equations by elimination	340
8-3	Solutions of systems of linear equations by substitution	346
8-4	Applications of systems of linear equations	351
8-5	Solving systems of linear inequalities by graphing	360
	Chapter 8 lead-in problem	363
	Chapter 8 summary	364
	Chapter 8 error analysis	364
	Chapter 8 critical thinking	364
	Chapter 8 review	365
	Chapter 8 cumulative test	366

Chapter 9 ■ Roots and Radicals



9-1	Principal roots	367
9-2	Product property for radicals	373
9-3	Quotient property for radicals	377
9-4	Sums and differences of radicals	383
9-5	Further operations with radicals	386
9-6	Fractional exponents	391
9-7	Equations involving radicals	395
	Chapter 9 lead-in problem	400
	Chapter 9 summary	400
	Chapter 9 error analysis	401
	Chapter 9 critical thinking	401
	Chapter 9 review	401
	Chapter 9 cumulative test	402

Chapter 10 ■ Solutions of Quadratic Equations



10-1	Solutions of quadratic equations by extracting the roots	404
10-2	Solutions of quadratic equations by completing the square	409
10-3	Solutions of quadratic equations by the quadratic formula	416
10-4	Complex solutions to quadratic equations	424
10-5	The graphs of quadratic equations in two variables—quadratic functions	432
	Chapter 10 lead-in problem	444
	Chapter 10 summary	445
	Chapter 10 error analysis	445
	Chapter 10 critical thinking	446
	Chapter 10 review	446
	Final examination	447
	Appendix Answers and Solutions	449
	Index	505

Index

A

Abscissa of a point, 281
 Absolute value, 31
 Addition
 of decimals, 16
 of fractions, 7–9, 239
 of rational expressions, 239, 246
 Addition, identity element of, 35
 Addition, properties of
 associative, 38
 commutative, 35
 identity, 35
 rational expressions, 239, 246
 Addition and subtraction property of equality, 88, 340
 Addition and subtraction property of inequalities, 115
 Addition of
 algebraic expressions, 80–81
 like radicals, 384–85
 more than two real numbers, 42
 rational expressions, 239, 246
 two negative numbers, 36
 two numbers with different signs, 36–37
 two positive numbers, 34
 two real numbers, 38
 Additive inverse, 37–38
 Algebraic expression, 67
 Approximately equal to, 19, 29, 369
 Associative property of
 addition, 38
 multiplication, 49
 Axes, 280
 Axis of symmetry, 437

B

Base, 56, 127
 Binomial, 68
 Binomial, square of, 135, 410
 Boundary line, 316
 Braces, 25, 42
 Brackets, 42

C

Cantor, Georg, 25
 Coefficient, 68
 Common denominator, 7
 Commutative property of
 addition, 35
 multiplication, 46
 Completely factored form, 157

Completing the square, 411–13
 Complex fractions, 253
 primary denominator, 253
 primary numerator, 253
 secondary denominators of, 253
 simplifying, 253–55
 Complex numbers, 425
 addition of, 425
 multiplication of, 426
 rationalizing the denominator of, 427
 Components of an ordered pair, 277
 Compound inequality, 114
 Conjugate factors, 388
 Consistent system of equations, 337
 Constant, 67
 Coordinate, 29, 281
 Coordinates of a point, 281
 Counting numbers, 26

D

Decimal number, 14
 fraction, 14
 point, 14
 repeating, 19, 27
 terminating, 27
 Decreasing, 29
 Degree of a polynomial, 69, 186
 Denominator
 of a fraction, 2
 least common, 42
 of a rational expression, 203
 Denominator, rationalizing a, 427, 480, 482
 Dependent system, 337
 Descending powers, 69, 163
 Difference of two squares, 136
 Discriminant, 429
 Distance-rate-time, 266, 352
 Distributive property of
 multiplication over addition, 79
 Dividend, 51
 Division
 monomial by monomial, 212
 polynomial by monomial, 212
 polynomial by polynomial, 213–16
 property of rational expressions, 234
 of rational expressions, 235
 Division, definition of, 51
 Division by zero, 53
 Division involving zero, 53
 Division of
 decimals, 17
 fractions, 5, 235
 like bases, 140
 two or more real numbers, 52
 two real numbers, 52

Divisor, 51
 Domain
 of a rational expression, 204–5
 of a relation, 323

E

Element of a set, 25
 Elimination, method of solving systems of linear equations, 340–44
 Empty set, 260, 398
 Equality, properties of
 addition and subtraction, 88
 multiplication and division, 94
 squaring, 396
 symmetric, 89
 Equation, parts of an, 86
 Equation of a line, 305
 Equations
 conditional, 87
 diagram of, 86
 equivalent, 87
 first-degree, 87
 graph of, 282–83, 289–94
 identical, 87
 linear, 87, 276
 literal, 104
 quadratic, 186, 404–31
 radical, 395–96
 rational, 258
 system of, 335
 Equivalent rational expressions, 245–46
 Evaluating rational expressions, 203–4
 Evaluation, 72
 Expanded form, 56, 127
 Exponent, 56, 127
 Exponential form, 56, 127
 Exponents, properties and definitions of
 definition, 127
 fraction, 139
 fractional, 392
 group of factors to a power, 129
 negative exponents, 141
 power of a power, 130
 product, 128
 quotient, 140
 zero as an exponent, 142
 Extracting the roots, 405–6
 Extraneous solutions, 260, 396

F

Factor, 3, 46
 Factor, greatest common, 156
 Factored form, completely, 157

Factoring

- common factors, 156
- difference of two cubes, 179–80
- difference of two squares 175–76
- four-term polynomials, 159
- by inspection, 170–74
- perfect square trinomials, 177–78
- strategy, 184
- sum of two cubes, 181–82
- trinomials, 162–64, 166–74

FOIL, 135**Formulas, 74, 104****Four-term polynomials, 159****Fraction, 2**

- complex, 253
- improper, 2
- proper, 2

Fraction exponents, 392**Fraction to a power, 139****Function, 324**

- domain of a, 325
- range of a, 325

Fundamental principle of rational expressions, 207**G****Graph**

- of a linear equation, in two variables, 282–83, 289–94
- of a point, 29, 359
- a quadratic equation, 432–40
- of systems of equations, 336–37

Graphing

- a linear equation in two variables, 289–94
- linear inequalities in two variables, 315–19
- quadratic functions, 434–39
- systems of linear equations, 336–37
- systems of linear inequalities, 360–61

Greatest common factor, 155–56**Grouping symbols, 42, 82****Grouping symbols, removing, 82****Group of factors to a power, 129****H****Half-planes, 316****Horizontal line, 292–93**

- slope of, 300

I**Identical equation, 87****Identity, 87****Identity element**

- of addition, 35
- of multiplication, 47

Inconsistent system, 337**Increasing, 29****Independent system, 337****Indeterminate, 53****Index, 370****Inequalities**

- compound, 114

linear, 113

- strict, 30
- in two variables, 315–19
- weak, 30

Inequalities, properties of

- addition and subtraction, 115
- multiplication and division, 115

Inequality symbols, 30, 113**Integers, 26****Intercepts, x- and y-, 290****Inverse property**

- additive, 38
- multiplicative, 94

Irrational numbers, 27, 368**L****Least common denominator (LCD), 7, 242****Least common multiple (LCM), 242****Like bases, 128****Like radicals, 383****Like terms, 80****Linear equation, 87****Linear equations in two variables, 276**

- graphs of, 289–93
- systems of, 335

Linear inequalities in two variables, 315

- graphs of, 315–19

Linear inequality, 113**Lines**

- equation of, 305
- horizontal, 300
- parallel, 309
- perpendicular, 310
- point-slope form, 306
- slope-intercept form of, 307
- slope of, 297–302
- standard form of, 305
- vertical, 301

Literal equation, 104**M****Mathematical statement, 86****Member of an equation, 86****Member of a set, 25****Mixed number, 5****Monomial, 68****Multinomial, 69****Multiple, least common, 242****Multiplication**

- of decimals, 16
- of fractions, 4
- identity element of, 47
- of rational expressions, 233
- symbols for, 46

Multiplication, properties of

- associative, 49
- commutative, 46
- fractions, 4, 232
- identity, 47

- rational expressions, 233

Multiplication and division property for inequalities, 115**Multiplication and division property of equality, 94****Multiplication of**

- like bases, 128
- monomial and a multinomial, 133
- monomials, 130
- multinomials, 134
- n th roots, 374
- square roots, 373
- two negative numbers, 47
- two numbers with different signs, 47
- two or more real numbers, 48
- two positive numbers, 46
- two real numbers, 48

Multiplicative inverse, 94**N****Natural numbers, 26****Negative exponents, 141****Negative integers, 26****Negative of, 31** **n th root**

- addition of, 383–85
- definition of, 370
- of a fraction, 378, 380
- rationalizing, 379–81, 388

Number line, 29**Numerator, 2, 203****Numerical coefficient, 68****O****Open sentence, 86****Opposite of, 31****Ordered pairs, first component of, 277****Ordered pairs, second component of, 277****Ordered pairs of numbers, 277****Order of operations, 57****Order relationships, 30****Ordinate of a point, 281****Origin, 29, 280****of a rectangular coordinate plane, 280****P****Parabola, 434**

- axis of symmetry of a, 437
- graphing a, 434–39
- intercepts of a, 435
- vertex of a, 436–37

Parallel lines, 309**Parentheses, 42****Percent, 19–20****Percentage, 20–21****Perfect cube, 180****Perfect square, 175–76****Perfect square integer, 368****Perfect square trinomial, 135, 409****Perimeter, 10, 74****Perpendicular lines, 310****Pi (π), 28****Point**

- abscissa of a, 281
- graph of a, 281
- ordinate of a, 281

Point-slope form, 306**Polynomial, 68**

Polynomial, degree of, 69, 186
 Positive integers, 26
 Power of a power, 130
 Prime factor form, 3
 Prime number, 3
 Prime polynomial, 164
 Principal n th root, 370
 Principal square root, 368
 Product, 3, 46
 Product of rational expressions, 233
 Product property of exponents, 128
 Product property of radicals, 374
 Product property of square roots, 373
 Properties of real numbers, 56
 Proportion, 221
 property of, 221
 terms of, 221
 Pythagorean Theorem, 369

Q

Quadrants, 280
 Quadratic
 equations, 186, 404–31
 formula, 417–19
 standard form of, 187, 404
 Quotient, 5, 51
 of two polynomials, 212–16
 Quotient property of exponents, 140
 Quotient property of radicals, 378, 380

R

Radical equation, 395–96
 Radicals, properties of
 product, 373, 374
 quotient, 378, 380
 Radical symbol, 368
 Radicand, 368
 Range of a relation, 323
 Ratio, 219
 Rational equation, 258
 application, 265–67
 in more than one variable, 261
 Rational expression, 202
 Rational expressions
 applications of, 265–67
 completely reduced, 208–10
 definition of, 202
 denominator of a, 203
 difference of, 239, 246
 domain of, 204
 fundamental principle of, 207
 least common denominator of, 242
 numerator of, 203
 product of, 233
 quotient of, 235
 sum of, 239, 246
 Rationalizing a denominator, 379–81, 388, 427
 Rational number, 27
 Real number, 28
 Real number line, 29
 Reciprocal, 5, 94
 Rectangular coordinate plane, 280
 Reducing to lowest terms, 4, 208

Relation, 323
 domain of a, 323
 range of a, 323
 Remainder, 5
 Repeating decimal, 19
 Root, 86
 Root, extraneous, 260

S

Scientific notation, 148
 computation using, 149
 standard form of, 149
 Set, 25
 Set symbolism, 25
 Signed numbers, 34
 Similar terms, 80
 Simple interest, 112
 Simplifying rational expressions, 207
 Slope, 297–98
 definition of, 298–99
 of a horizontal line, 300
 of parallel lines, 309
 of perpendicular lines, 310
 undefined, 300–1
 of a vertical line, 301
 Slope-intercept form, 307
 Solution, 86, 276
 extraneous, 260
 Solution set, 86, 187
 of a linear equation in two variables, 277
 Solutions of quadratic equations
 by completing the square, 411–13
 complex solutions of, 427
 by extracting the roots, 405–6
 by factoring, 404–5
 by quadratic formula, 418–19
 Special products, 135–37
 Square of a binomial, 135
 Square root, 367
 Square root property, 405
 Standard form of a quadratic equation, 404
 Standard form of the equation of a line, 305
 Statement, mathematical, 86
 Straight line, 289
 Strict inequality, 30
 Subscripts, 75
 Subset, 26
 Substitution property, 72
 method of solving a system of linear equations, 346–48
 Subtraction, definition of, 41
 decimals, 16
 Subtraction, of rational expressions, 239
 Subtraction of
 fractions, 7–9, 239
 more than two real numbers, 42
 two real numbers, 41
 Symbols
 absolute value, 31
 braces, 25, 42
 brackets, 42
 is an element of, 26
 is approximately equal to, 19
 is a subset of, 26
 is greater than, 30
 is greater than or equal to, 30

is less than, 30
 is less than or equal to, 30
 multiplication dot, 46
 null set, 260
 parentheses, 42
 Symmetric property of equality, 89
 Symmetry, 31
 Systems of linear equations, 335
 applications of, 351–55
 consistent and independent, 337–38
 dependent, 337–38
 graphing of, 336–37
 inconsistent, 337–38
 linear, 335
 solution by elimination, 340–44
 solution by substitution, 346–48
 Systems of linear inequalities, 360–61

T

Term, 67
 Trinomial, 68
 Trinomial, perfect square, 135, 409

U

Undefined, 53
 Undefined slope, 301
 Undirected distance, 31

V

Variable, 29, 67
 Vertex of a parabola, 436–37
 Vertical line, 293
 slope of, 301

W

Weak inequality, 30
 Whole numbers, 26

X

x -axis, 280
 x -intercept, 290, 435

Y

y -axis, 280
 y -intercept, 290, 435

Z

Zero, division involving, 53
 Zero as an exponent, 142
 Zero factor property, 47
 Zero product property, 187



Need more money for college expenses?

The CLC Private LoanSM can get you up to
\$40,000 a year for college-related expenses.

Here's why the CLC Private LoanSM is a smart choice:

- ✓ Approved borrowers are sent a check within four business days
- ✓ Get \$1000 - \$40,000 each year
- ✓ Interest rates as low as prime + 0% (8.66% APR)
- ✓ Quick and easy approval process
- ✓ No payments until after graduating or leaving school

CLICK HERE

or call **800.311.9615.**

*We are available 24 hours
a day, 7 days a week.*